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Efficient symmetry-based screening strategy to speed up randomized circle-detection

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ABSTRACT

Randomized approaches for circle detections are often used for the advantages of less computational time and memory requirements. However, randomized approaches involve examining a large number of candidate circles and may not be suitable for real-time applications. In this paper, a screening strategy based on the symmetric property of the circle is adopted to select the promising candidates for further investigation, resulting in substantial reduction in the computational time while maintaining the accuracy. Empirical results show that, under the same accuracy level, the proposed symmetry-based method achieves the improvement ratios of 40%–90% on the execution-time when compared to four state-of-the-art randomized methods.

1. Introduction

Circle detection is an important and fundamental operation in image processing. As is true for most algorithms, the issues of robustness and computational efficiency are of major concern. The developed circle-detection methods can generally be classified into two categories — deterministic and nondeterministic methods. Deterministic methods aim for detection accuracy while nondeterministic methods are more concerned about the computational efficiency.

Most deterministic circle-detection methods are based on Hough transform (HT) technique (Hough, 1962; Duda and Hart, 1972). Hough transform technique counts the number of edge pixels falling on a specific circle which is characterized by the center coordinates and radius. Hough transform technique creates a 3-dimensional accumulator array where each entry is indexed by the center coordinates and radius of a possible circle to accumulate the number of edge pixels falling on that possible circle. The possible circle with large number of edge pixels, i.e. votes, is declared as a detected circle. The exhaustive searching process in HT-based methods results in superior detection accuracy but suffers from large computational cost and memory requirement due to the use of accumulator arrays. To alleviate the problems of computational cost and memory requirement, geometric properties are often used to reduce the search space for possible circles. Kimme et al. (1975) used the gradient information of edge pixels to identify the centers of possible circles. Yip et al. (1992) explored the parallel tangent lines of edge pixels to identify the possible circles. Using the property that the circle center lies on the perpendicular bisector of a chord, Ho and Chen (1995) used the intersections of the bisectors corresponding to horizontal and vertical chords as the centers of possible circles. Ioannou et al. (1999) and Kim and Kim (2001) identified bisectors corresponding to the possible chords and the points with large number of bisectors passing through were used as the centers of the possible circles. Illingworth and Kittler (1987) adopted the coarse-to-fine principle by first identifying the possible circles based on the image with lower resolution and then the image with higher resolution to efficiently manage the size of the accumulator array. Chiu and Liaw (2005) only constructed the possible circles based on three edge pixels such that the two side pixels are of approximately equal distance from the middle pixel.

However, these improved HT-based circle-detection algorithms, either by using the geometric properties or by managing the size of

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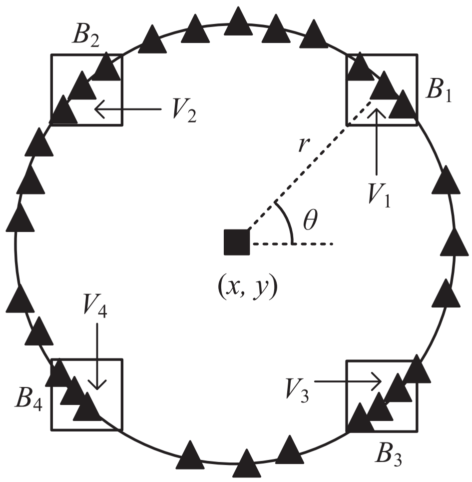

Fig. 1. Edge sets used for testing the symmetry feature of the possible circle *C*.

the accumulator array, still search over a large search space and are not suitable for real-time applications. Instead of examining all possible circles, nondeterministic methods only examine the possible circles randomly generated and stop searching when the number of detected circles stops growing. Nondeterministic circle detection methods first randomly generate a possible circle, then verify if the possible circle is a legitimate candidate circle, and finally determine if the candidate circle is a detected circle by a voting process.

In order to meet the real-time requirements, the randomized or nondeterministic circle-detection methods (Xu et al., 1990; Chen and Chung, 2001; Chung and Huang, 2007; Lu and Tan, 2008) have been developed to speed up the computation. For randomized circle-detection methods, a possible circle is first created using three of the four pixels randomly selected and the possible circle becomes a legitimate candidate circle if the fourth pixels falls on the possible circle. A voting process which counts the number of edge pixels falling on the legitimate candidate circle is used to determine if the candidate circle is a detected circle.

Since the voting process often involves examining a large number of edge pixels, the computational overhead heavily depends on the number of candidate circles. Furthermore, since the randomized circle detection methods usually generate a large number of candidate circles, a screening scheme which sifts only the promising candidates from the possible circles for voting process may substantially reduce the computational time.

This paper proposes a screening strategy which sifts, based on the symmetric property of the circle, the promising candidate circles from the possible circles created by the randomized approaches. Since the screening strategy can successfully discard a large amount of unpromising candidate circles, the proposed method substantially reduces the computational time while preserving the detection accuracy. Applying the proposed screening strategy to sift promising candidate circles from the possible circles obtained by the existing four state-of-the-art randomized methods (Xu et al., 1990; Chen and Chung, 2001; Chung and Huang, 2007; Lu and Tan, 2008), we present four new and fast symmetry-based methods for circle detection. Experimental

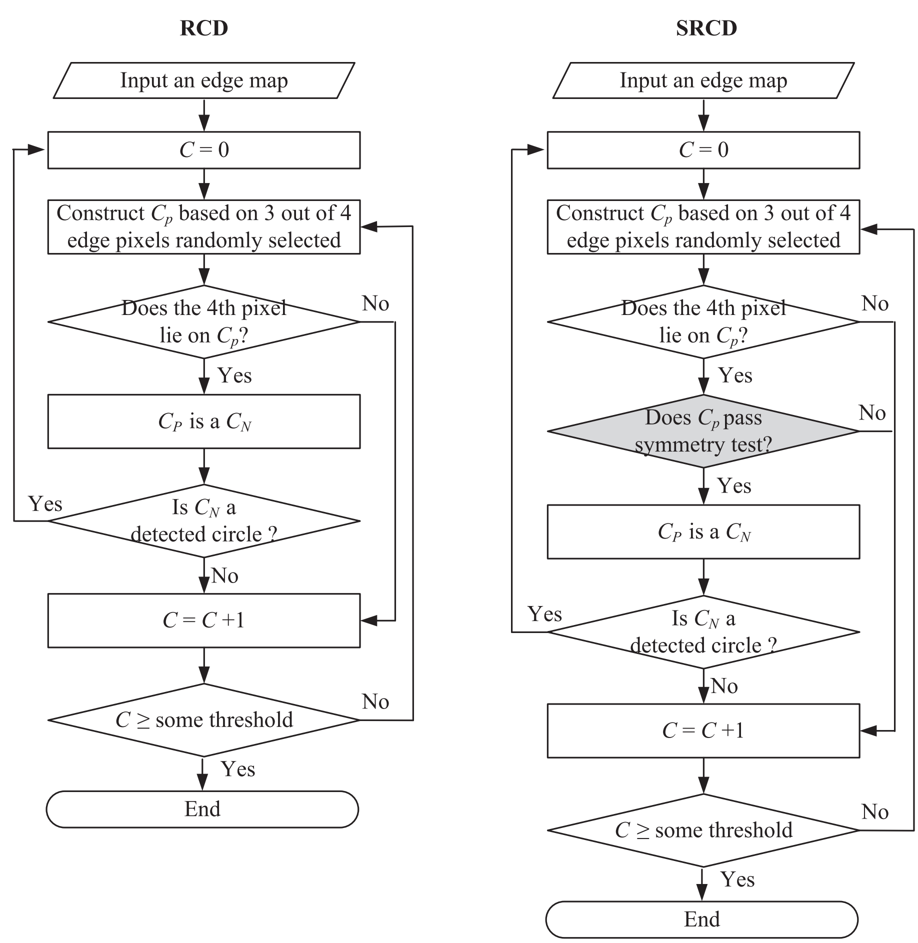

Fig. 2. Flowcharts of the RCD and the proposed SRCD.

results show that, while maintaining the same accuracy level, the proposed symmetry-based methods can achieve the improvement ratios of 40%–90% on the execution-time when compared to the existing four randomized methods.

The rest of this paper is organized as follows. Section 2 introduces the proposed symmetry-based screening strategy. Section 3 presents the four new circle detection methods and the empirical results. Concluding remarks are given in Section 4.

2. The proposed symmetry-based screening strategy

This section presents the proposed screening strategy which makes use of the symmetric property of the circle to identify the promising candidates for further investigation. Suppose the possible circle, as shown in Fig. 1, is obtained by some randomized method and is centered at (x, y) with radius r. Considering the rectangle B_1 in Fig. 1, let (x, y) be the origin of the coordinate system and $(r, \theta + \pi/36)$ and $(r, \theta - \pi/36)$ denote the polar coordinates of the left-upper and right-lower corners of rectangle B_1. Rectangles B_2 and B_3 are symmetric to B_1 about the y-axis and the x-axis, respectively, while rectangle B_4 is symmetric to B_1 about the origin. Let V_i denote the set of edge pixels which are represented as triangles in Fig. 1, contained in rectangle B_i for $i = 1, \ldots, 4$. For the symmetry test to be useful, we require that the number of edge pixels contained in the rectangle should reach some threshold. Set V_i is defined to be valid if the number of edge pixels in V_i is greater than or equal to the threshold $(0.6)r(2\pi/36)$ where $r(2\pi/36)$ denotes the arc length covered by each of the four rectangles and '0.6' denotes the allowable percentage of the arc length. Since at least two valid sets of edge pixels are required to perform the symmetry test, the parameter θ for rectangle B_1 is determined such that at least two valid sets of edge pixels exist. For the four rectangles to be uniformly distributed over the circumference of the possible circle, we search for θ along the list of $\pi/4, \pi/4 + \Delta, \pi/4 - \Delta, \pi/4 + 2\Delta, \ldots, \pi/4 - 4\Delta$ with $\Delta = \pi/18$. The search process for θ stops once at least two valid sets of edge pixels are found.

Given a possible circle generated by some randomized method, suppose θ is determined such that m ($\geqslant 2$) valid edge sets $V_1, V_2, \ldots, V_m$ are found. The proposed symmetry test measures the symmetry level of the possible circle based on the pairs of valid edge sets. Consider a pair of valid edge sets (V_k, V_ℓ), $k < \ell$, which can be symmetric to each other about the x-axis, y-axis, or the origin. To measure the symmetry level between valid edge sets V_k and V_ℓ, first we need to perform mirror transformation on each of the edge pixels in V_ℓ. Denote by $\widetilde{V}_\ell$ the set of mirrored pixels which are mirror-transformed from the edge pixels in V_ℓ, depending on the relationship between V_k and V_ℓ. For example, if V_k and V_ℓ are symmetric about the origin then $\widetilde{V}_\ell$ is symmetric to V_ℓ about the origin. The symmetry level between V_k and V_ℓ is measured by the Hausdorff distance (Munkres, 1999) between V_k and $\widetilde{V}_\ell$ which can be expressed as

$$H(V_k, \widetilde{V}_\ell) = \max(h(V_k, \widetilde{V}_\ell), h(\widetilde{V}_\ell, V_k)) \quad (1)$$

where

$$h(V_1, V_2) = \max_{p \in V_1} \min_{q \in V_2} (||p - q||) \quad (2)$$

with $||p - q||$ denoting the Euclidean distance between pixels p and q. In mathematics, the Hausdorff distance measures the distance

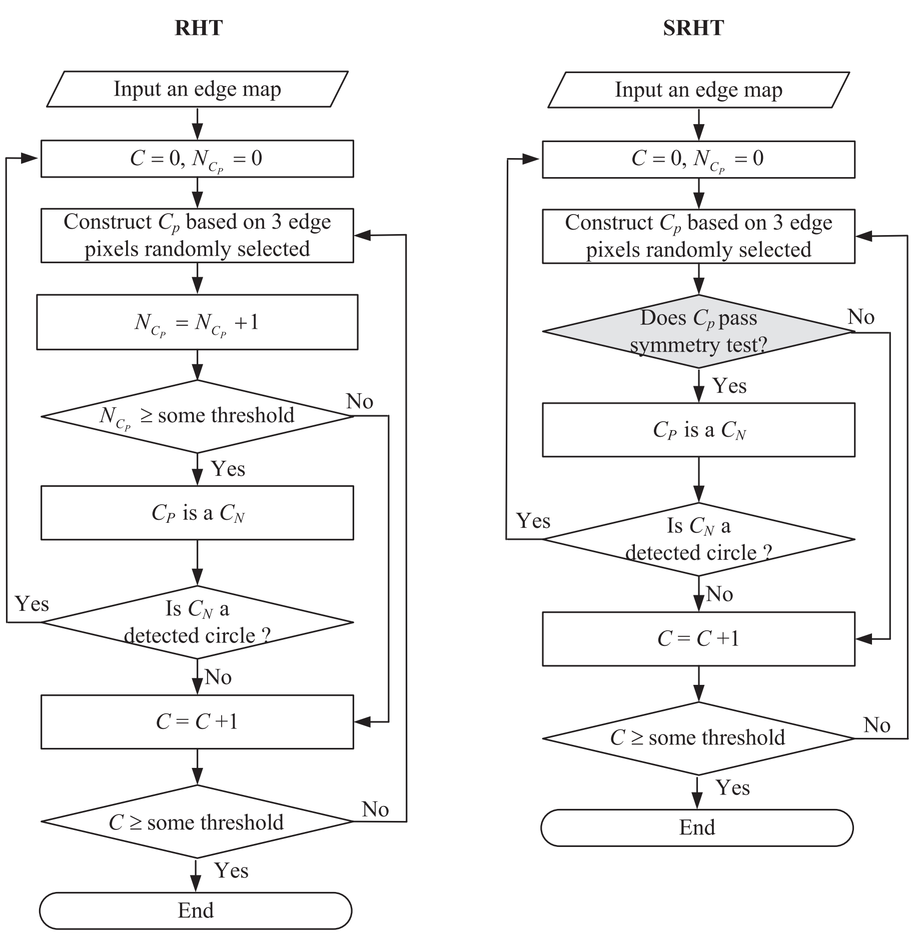

Fig. 3. Flowcharts of the RHT and the proposed SRHT.

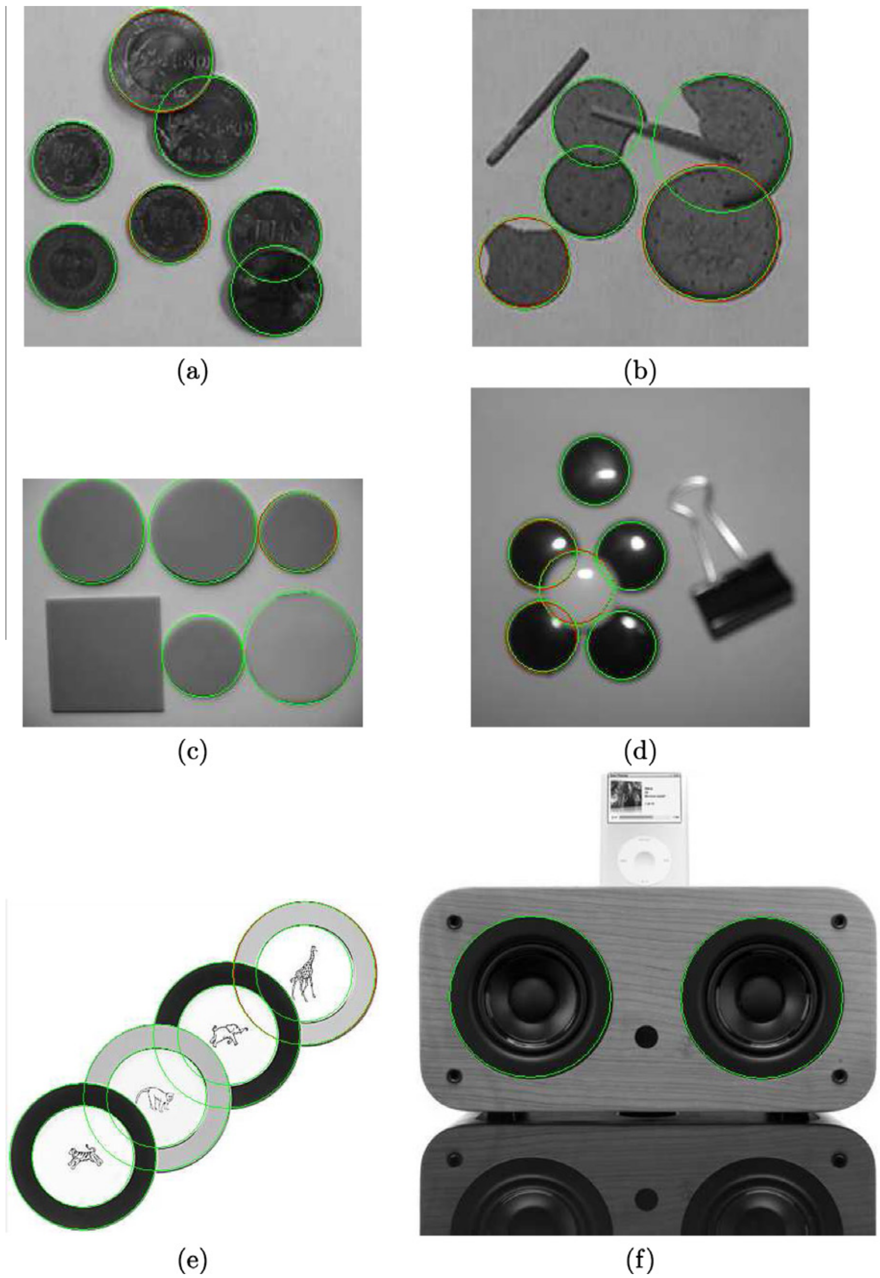

Fig. 4. Six test images and the detected circles by RCD and SRCD.

Table 1
Performance comparisons based on the averages of $\#C_P$, $\#C_N$, execution-time in milliseconds, and accuracy.

	$\#C_P$	$\#C_N$	Execution time	Accuracy
SRCD	37447	130	44	1.21
(RCD)	(37605)	(1709)	(112)	(1.26)
SLRCD	37877	129	39	1.34
(LRCD)	(37925)	(1766)	(66)	(1.36)
SRHT	24241	1550	206	1.08
(RHT)	(73243)	(4572)	(426)	(1.07)
SIRHT	18678	38	74	0.90
(IRHT)	(406618)	(6)	(733)	(0.89)

between two subsets of a metric space. Informally, two sets are close in the Hausdorff distance if every point of either set is close to some point of the other set. Smaller Hausdorff distance between V_k and $\widetilde{V}_\ell$ indicates that the edge sets V_k and V_ℓ have higher symmetry level. The possible circle becomes a candidate circle and will be further investigated by the voting process only when the symmetry level of some pair of valid edge sets is above a prespecified threshold.

SYMMETRY_TEST: *Given a possible circle generated from some randomized method, determine θ such that there exist at least two valid edge sets, and for each pair of valid edge sets, perform mirror transformation and calculate the Hausdorff distance. The possible circle becomes a candidate circle and will be further investigated by the voting process when there exists one pair of valid edge sets with satisfactory symmetry level.*

3. Proposed symmetry-based methods for circle detection

In this section, the proposed symmetry-based screening strategy is incorporated with four state-of-the-art randomized methods for circle detection — the randomized circle detection (RCD) method (Chen and Chung, 2001), the lookup table-based RCD (LRCD) method (Chung and Huang, 2007), the randomize Hough transform (RHT) method (Xu et al., 1990), and the iterative RHT (IRHT) method (Lu and Tan, 2008).

For the RCD method, three of the four randomly selected edge pixels are randomly selected to construct a possible circle C_P. If the fourth pixel is close to the possible circle, regard the possible circle as a candidate circle C_N which will be further investigated by a voting process. The candidate circle is declared as a detected circle if the number of edge pixels, counted by a voting process, is greater than some threshold. The LRCD method differs from the RCD method in that it speeds up the voting process by a table lookup technique which determines if a specific edge pixel lies on the considered candidate circle. Both RCD and LRCD methods terminate when the number of trials failed to find a candidate circle or to declare a detected circle reaches some threshold.

In addition to using the fourth pixel to determine the candidate circles in the RCD and LRCD methods, the proposed symmetry test is used to screen the promising candidate circles, leading to substantial reduction in the computational time in the subsequent voting process. The methods which apply the symmetry test on the RCD and LRCD methods are named by SRCD and SLRCD respectively. The algorithms corresponding to the RCD method and the proposed SRCD method are given in Fig. 2.

Hough transform method achieves high accuracy but suffers from formidable computational cost due to the exhaustive search in the 3-dimensional parameter space characterized by the center coordinates and radius of a circle. The 3-dimensional parameter space is usually implemented as a 3-dimensional accumulator array with size proportional to the image size by the length of the diagonal line of the image. Instead of investigating every possible circle, randomized Hough transform (RHT) method randomly generates possible circles based on three edge pixels randomly selected. A possible circle is declared as a candidate circle if the number of its appearance, recorded in the accumulator array, reaches some threshold. The voting process is then used to determine if the candidate is a detected circle. Instead of sending the candidate circle with the maximum number of appearance, generated by the RHT method, to the voting process, iteratively apply the RHT method on the region which frames the candidate circle to identify the circle with the maximum number of appearance. The iteration terminates once there exists no significant difference between the circles identified in adjacent iterations and the circle identified in the last iteration is regarded as a candidate circle for voting process. This iterative RHT method, named by IRHT, generally generates more possible circles and fewer candidate circles for voting process. Both RHT and IRHT methods terminate when the number of trials failed to find a candidate circle or to declare a detected circle reaches some threshold.

Instead of using the accumulator array to record the appearing frequency of each possible circle, any possible circle passing the proposed symmetry test is regarded as a candidate for voting process. This method, named by SRHT, does not require the accumulator array and usually takes less computational time. As in each iteration for IRHT, the proposed symmetry test, instead of counting the appearing frequency, is used to identify the circle. This method, named by SIRHT, usually generates much fewer number of possible circles than the IRHT method. The algorithms corresponding to the RHT method and the proposed SRHT method are given in Fig. 3.

4. Empirical study

For demonstrating the effectiveness of the proposed SRCD and SLRCD methods, execution-time and detection accuracy are evaluated based on six test images, as shown in Fig. 4. For each test image, the ground-truth parameters of center and radius are determined by the traditional Hough transform methods Hough (1962); Duda and Hart (1972). Since each randomized method may report different number of detected circles, the replication reporting the correct number of detected circles is regarded as a successful replication. The performance measures of accuracy and computational time are computed as the averages over 1000 successful replications.

Consider the test image with k ground-truth circles, the accuracy associated with the detected circle with center (x', y') and radius r' in a successful replication is computed as

$$\min_{1\leqslant j\leqslant k}\sqrt{(x'-x_j)^2+(y'-y_j)^2+(r'-r_j)^2},$$

where (x_j, y_j) and r_j denote, respectively, the center and radius of the jth ground-truth circle for $j = 1, \ldots, k$. The accuracy corresponding to each randomized method is the grand average of the accuracy over the number of ground-truth circles, 1000 successful replications, and 6 test images. Similarly, the average execution time for each method is the average over 100 successful replications and 6 test images. In addition to the average accuracy and average execution time, Table 1 also gives the number of constructed possible circles $\#C_P$ and the number of candidate circles $\#C_N$. All experiments are performed on the Intel CPU E8400 Processor with 3 GHz and 2 GB RAM.

Based on the empirical results, the following general conclusions are obvious:

1. The proposed symmetry-based screening strategy can speed up the existing state-of-the-art circle detection methods while preserving the detection accuracy, indicating that the strategy can successfully remove the unpromising candidate circles.
2. The proposed symmetry-based screening strategy can substantially reduce the number of candidate circles, indicating that the symmetric property of a circle is an important criterion in differentiating a circle.
3. Since the proposed symmetry-based test is used for directly screening the candidate circles, no accumulator array is required, leading to little memory requirement.
4. When compared to the RCD and LRCD methods, the proposed strategy substantially reduces the number of candidate circles.
5. When compared to the RHT and IRHT methods, since the proposed screening strategy is used to directly differentiate if a possible circle is a candidate circle, no accumulator array is used to accumulate the appearing frequencies of the possible circles, leading to small number of possible circles.

For subjective evaluation of the proposed screening strategy, Fig. 4 shows the detected circles, averaged over 1000 replications, by the RCD and SRCD methods. The red and green circles denote, respectively, the detected circles by the RCD and SRCD methods. In the case where a red circle is overlapped with a green circle, a green circle is used to represent the detected circle. Empirical results indicate that the proposed screening strategy will not deteriorate the accuracy. Similar results were observed in other randomized circle detection methods.

5. Conclusion

This paper proposes a symmetry-based screening strategy to speed up the four existing state-of-the-art randomized circle detection methods while preserving the detection accuracy. The proposed symmetry-based screening strategy substantially reduces the number of candidate circles without deteriorating the detection accuracy, indicating that the symmetry of the circle indeed serves as an efficiently discriminating criterion. Since the voting process for each candidate circle involves examining a large number of edge pixels, reducing the number of candidate circles substantially reduces the execution time. Furthermore, since the symmetry test directly sifts the candidate circles from the possible circles without using the accumulator array to record the appearing frequencies of the possible circles, the proposed symmetry-based method requires little memory space. Although the proposed method can substantially reduce the number of candidate circles, the computational time does not reduce proportionally, indicating that designing an efficient symmetry test may lead to significant reduction on the computational time.

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