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Novel fractal image encoding algorithm using normalized one-norm and kick-out condition

Hsiu-Niang Chen^{a,b,1}, Kuo-Liang Chung^{b,*,2}, Jian-Er Hung^b

^aDepartment of Information Management, Vannung University of Science and Technology, No. 1, Vannung Road, Shuiwei, Chungli 320, Taiwan, ROC

^bDepartment of Computer Science and Information Engineering, National Taiwan University of Science and Technology, No. 43, Section 4, Keelung Road, Taipei 10672, Taiwan, ROC

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ABSTRACT

For fractal image encoding, based on a special measure called the one-norm of normalized block, this paper presents a novel kick-out method to discard impossible domain blocks in early stage for the current range block. It leads to speed up the encoding time. Since our proposed kick-out method is based on Jacquin's full search method, both methods need to search the whole image and the decoded image quality are the same. Based on five typical testing images, our proposed method has 22% execution time improvement ratio in average when compared with Jacquin's full search method. Combining our proposed method with Truong et al.'s DCT inner product method, Lai et al.'s kick-out method, or both methods, the encoding-time performance can be improved further.

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1. Introduction

Fractal image compression was first proposed by Barnsley [1] according to the contractive mapping fixed-point theorem and first realized by Jacquin [2] based on a partitioned iterated function system. In fractal image encoding process, for an input range block, searching the best matched domain block in a large domain pool is very time-consuming [3]. In order to alleviate this serious encoding-time problem, some efficient fractal encoding methods have been developed. These encoding algorithms include the partitioned-based approach [3–6], the domain pool selection approach [3,7], and the search strategy-based approach [4,8–21]. In 2008, there are three newly published works, Kovács [20] presented a classification-based fractal image encoding method. Wang and Wang [21] presented an improved no-search method to reduce the execution time requirement and increase the decoded image quality when compared to the quadtree partition scheme. Based on particle swarm optimization approach, Tseng et al. [19] presented a fractal encoding method to speed up the encoder 125 times faster with only 0.89 dB decay of image quality in compar-

ison to Jacquin's full search method. Without confusion, Jacquin's full search method is also called the full search method for simplicity.

Although many previous methods do reduce the computation effort significantly, they may suffer from the degradation of the reconstructed image quality when compared with the full search method. Note that although the full search method for fractal image encoding does search the whole image, it has some decoded image degradation when compared to the original input image. Recently, two computation-efficient fractal encoding methods were presented by Truong et al.'s DCT inner product method and Lai et al.'s method. Both methods have the same decoded image quality as in the full search method since their methods are also based on the full search method.

In this paper, based on a special measure called the one-norm of normalized block, we present a novel inequality which can discard redundant computations involved in the error-term calculation which is one of the kernel processes in the encoding process. That is, for each range block, our proposed kick-out method can discard impossible domain blocks as early as possible in the process for finding the best matched domain block. Because our proposed kick-out method is based on the full search method, the proposed method thus has the same decoded image quality as in the full search method. Based on five testing images, experimental results showed that when compared with the full search method, our proposed method has 22% execution time improvement ratio in average. In average, combining our proposed method with Truong

* Corresponding author. Tel.: +886 2 27376771.

E-mail address: k.l.chung@ntust.edu.tw (K.-L. Chung).

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et al.'s method has 56% execution-time improvement ratio; combining our proposed method with Lai et al.'s method has 45.5% execution-time improvement ratio; combining our proposed method with both methods has 69.5% execution-time improvement ratio.

The remainder of this paper is organized as follows. In Section 2, we survey the past three works, the full search method, Truong et al.'s DCT inner product method, and Lai et al.'s kick-out method. In Section 3, our proposed kick-out based method is presented. Experimental results are demonstrated in Section 4. Some concluding remarks are addressed in Section 5.

2. The past three works

In fractal image compression method, the input image f is first partitioned into two kinds of basic block units: the nonoverlapping range blocks, each with size $N = n \times n$, and the overlapping domain blocks, each with size $M = m \times m$. Suppose the number of range blocks is N_R , then the image f can be expressed by

$$f = \bigcup_{i=1}^{N_R} R_i. \quad (1)$$

The size of a domain block is usually four times that of a range block, i.e., $M = 4N = 2n \times 2n$. To encode a range block R , each domain block in the domain pool is scaled to the size of the range block. Each pixel value in the contracted domain block can be represented by the mean of the four neighboring pixel values in the original domain block. Assume that the number of domain blocks is N_D . The set of these N_D contracted domain blocks is denoted by $\{D_i | i = 1, \dots, N_D\}$.

2.1. The full search method by Jacquin

In order to find the best matched domain block in the large domain pool for an input range block R , the following error term should be minimized:

$$E(R, D_i) = \|R - (sD_i + oI)\|^2 \quad (2)$$

where the symbol $\|\cdot\|$ denotes the 2-norm operation, D_i is the contracted domain block under one of eight possible isometry transformations, I represents the constant block whose values are all ones, and s and o are the contrast and brightness offset parameters, respectively. The term $(sD_i + oI)$ is an intensity affine mapping to adjust the contrast and brightness of the block D_i . In order to determine the best matched domain block for an input range block, the eight isometry transformations consisting of four orientations and four reflections of each domain block must be considered.

Given a range block R and the corresponding domain block D , the linear least squares method can be used to determine the contrast and brightness offset parameters. Then, the two parameters s and o can be computed by

$$s = \frac{\langle R - \bar{r}I, D - \bar{d}I \rangle}{\|D - \bar{d}I\|^2} \quad \text{and} \quad o = \bar{r} - s\bar{d} \quad (3)$$

where \bar{r} and \bar{d} represent the mean intensity of the block R and D , respectively, and the symbol $\langle \cdot, \cdot \rangle$ denotes the inner product operation. In order to ensure the convergence in the iterated decoding process, the contrast parameter s should satisfy $|s| < 1$ [3]. Once s and o are obtained, the error $E(R, D)$ can be computed. By $o = \bar{r} - s\bar{d}$ in (3), the error $E(R, D)$ can be further simplified as follows:

$$E(R, D) = \|R - (sD + oI)\|^2 = \|R - \bar{r}I\|^2 - s^2\|D - \bar{d}I\|^2 = u - s^2v. \quad (4)$$

The domain block which results in the smallest error from (2) is selected as the best matched block and all the corresponding param-

eters are stored. The encoding parameters include s , o , the location of the best matched domain block, and the index denoting which isometry transformation is used. In the decoding process, the stored parameters are recursively applied to the initial image. Then, the original image will be reconstructed after a few iterations.

Suppose the input image f is with size 512×512 and each range block is with size 4×4 . The domain blocks are considered by sliding a window of size 8×8 in a given incremental step horizontally or vertically. If the incremental step is set to 8, we have $N_D = ((512 - 8)/8 + 1) \times ((512 - 8)/8 + 1) = 4096$ domain blocks. Therefore, given an input range block, we need to compute $4096 \times 8 = 32,768$ errors to obtain the best matched block. Since there are $N_R = (512/4) \times (512/4) = 16,384$ range blocks, based on full search method, we need to compute $536,870,912 (= 32,768 \times 16,384)$ errors to encode the input image and it is very time-consuming. That is why the encoding time needs to be improved.

2.2. The DCT inner product method by Truong et al.

In [8], Truong et al. proposed a DCT inner product method to reduce the number of isometry transformations from eight to two. The inner product operation in Eq. (3) can be calculated in the DCT domain as shown below:

$$\delta_k = \langle R - \bar{r}I, D_k - \bar{d}I \rangle = \langle \tilde{R}, \tilde{D}_k \rangle = \sum_{i=0}^7 \sum_{j=0}^7 \tilde{R}(i,j) \tilde{D}_k(i,j) \quad (5)$$

where \tilde{R} and \tilde{D}_k indicate the DCT coefficient matrices of $R - \bar{r}I$ and $D_k - \bar{d}I$ for the k th isometry transformation for $1 \leq k \leq 8$. Let $a_{ij} = \tilde{R}(i,j) \tilde{D}_1(i,j)$ and $b_{ij} = \tilde{R}(i,j) \tilde{D}_1(j,i)$, then by (5), it yields to

$$\begin{aligned} \delta_1 &= \sum_{i=0}^7 \sum_{j=0}^7 a_{ij}, & \delta_2 &= \sum_{i=0}^7 \sum_{j=0}^7 (-1)^i a_{ij} \\ \delta_3 &= \sum_{i=0}^7 \sum_{j=0}^7 (-1)^j a_{ij}, & \delta_4 &= \sum_{i=0}^7 \sum_{j=0}^7 (-1)^{i+j} a_{ij} \\ \delta_5 &= \sum_{i=0}^7 \sum_{j=0}^7 b_{ij}, & \delta_6 &= \sum_{i=0}^7 \sum_{j=0}^7 (-1)^j b_{ij} \\ \delta_7 &= \sum_{i=0}^7 \sum_{j=0}^7 (-1)^i b_{ij}, & \delta_8 &= \sum_{i=0}^7 \sum_{j=0}^7 (-1)^{i+j} b_{ij} \end{aligned} \quad (6)$$

From (6), instead of calculating eight isometry transformations, only two isometry transformations, $\delta_1 = \sum_{i=0}^7 \sum_{j=0}^7 a_{ij}$ and $\delta_5 = \sum_{i=0}^7 \sum_{j=0}^7 b_{ij}$, are needed to be computed and the other six isometry transformations can be obtained easily. The DCT inner product method has a good computation-saving effect and the same decoded image quality as in the full search method.

2.3. Lai et al.'s method

In [11], the authors proposed a fast fractal encoding method based on kick-out and zero contrast conditions. By (4), it is known $E(R, D) = u - s^2v$. Due to $|s| < 1$, it yields $E(R, D) \geq u - v$. Let $E_{L2}(R, D) = u - v$, we thus have

$$E(R, D) \geq E_{L2}(R, D). \quad (7)$$

For the input range block R , assume that D_{min} is the current minimum error. For any domain block D , if the kick-out condition

$$E_{L2}(R, D) \geq D_{min} \quad (8)$$

is held, it follows that $E(R, D) \geq D_{min}$. Therefore, the domain block D will not be the best matched block to the range block R and it can be kicked out immediately.

Besides (8), the zero contrast condition is also used to speed up the determination of the best matched domain block. From $E(R, D) = u - s^2v \geq 0$, we have

$$|s| \leq \sqrt{\frac{u}{v}} \quad (9)$$

If the contrast parameter $|s| \leq 0.03125$, s will be quantized to 0. On the other hand, when

$$\sqrt{\frac{u}{v}} < 0.03125, \quad (10)$$

s is set to 0. Thus, the corresponding error is given by

$$E(R, D) = u. \quad (11)$$

Lai et al.'s method also has a good computation-saving effect and has the same decoded image quality as in the full search method.

3. Our proposed kick-out based method

In this section, based on one-norm of normalized block measure, a new inequality is derived first and it can be used to discard the impossible domain blocks as early as possible in the search process. The proposed method also keeps the same decoded image quality as in the full search method. We will explain it later.

For an image block X , we denote its normalized block as $\hat{X} = (X - \bar{x}I) / \|X - \bar{x}I\|$. The one-norm of the normalized range block \hat{R} is defined as follows:

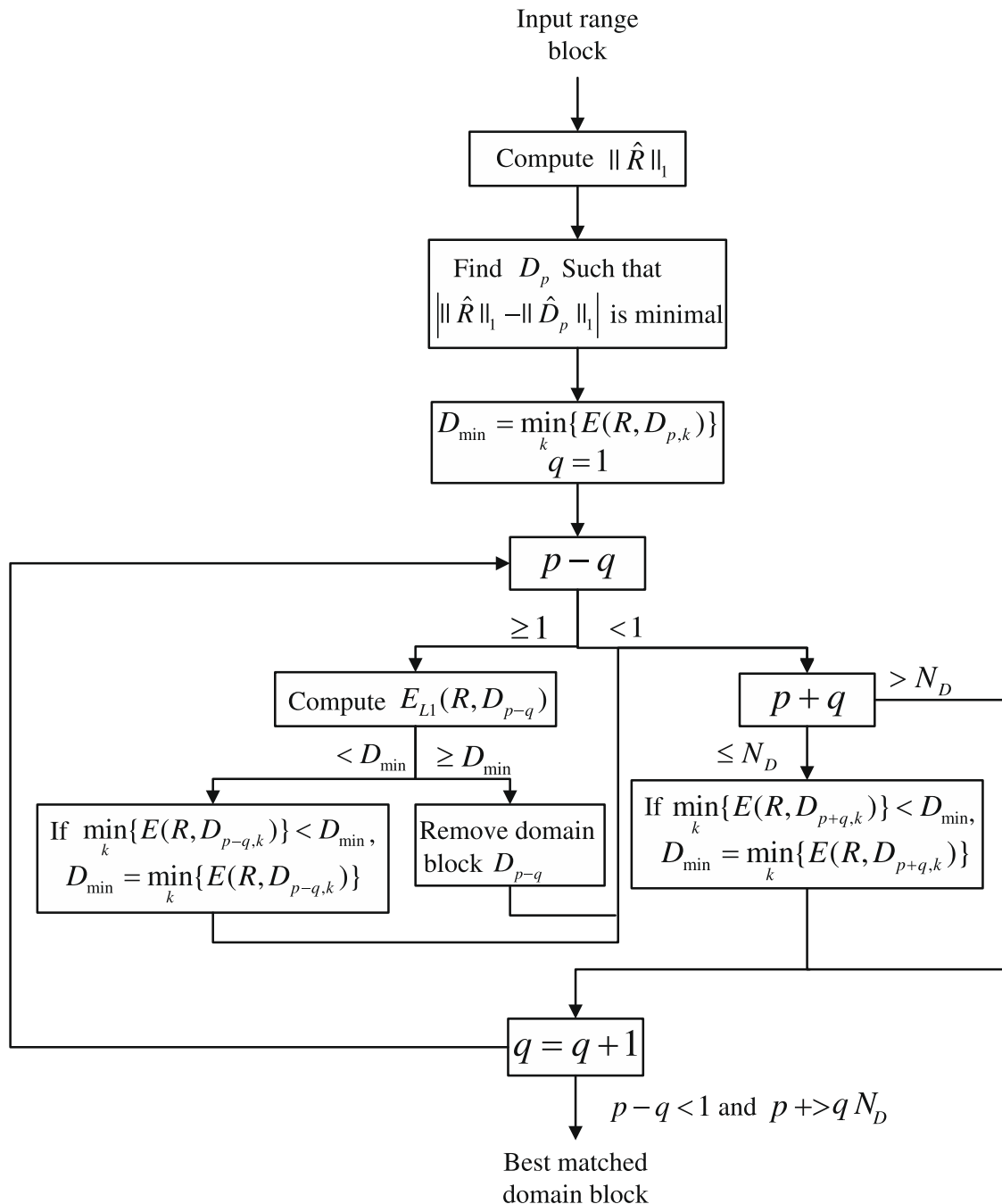
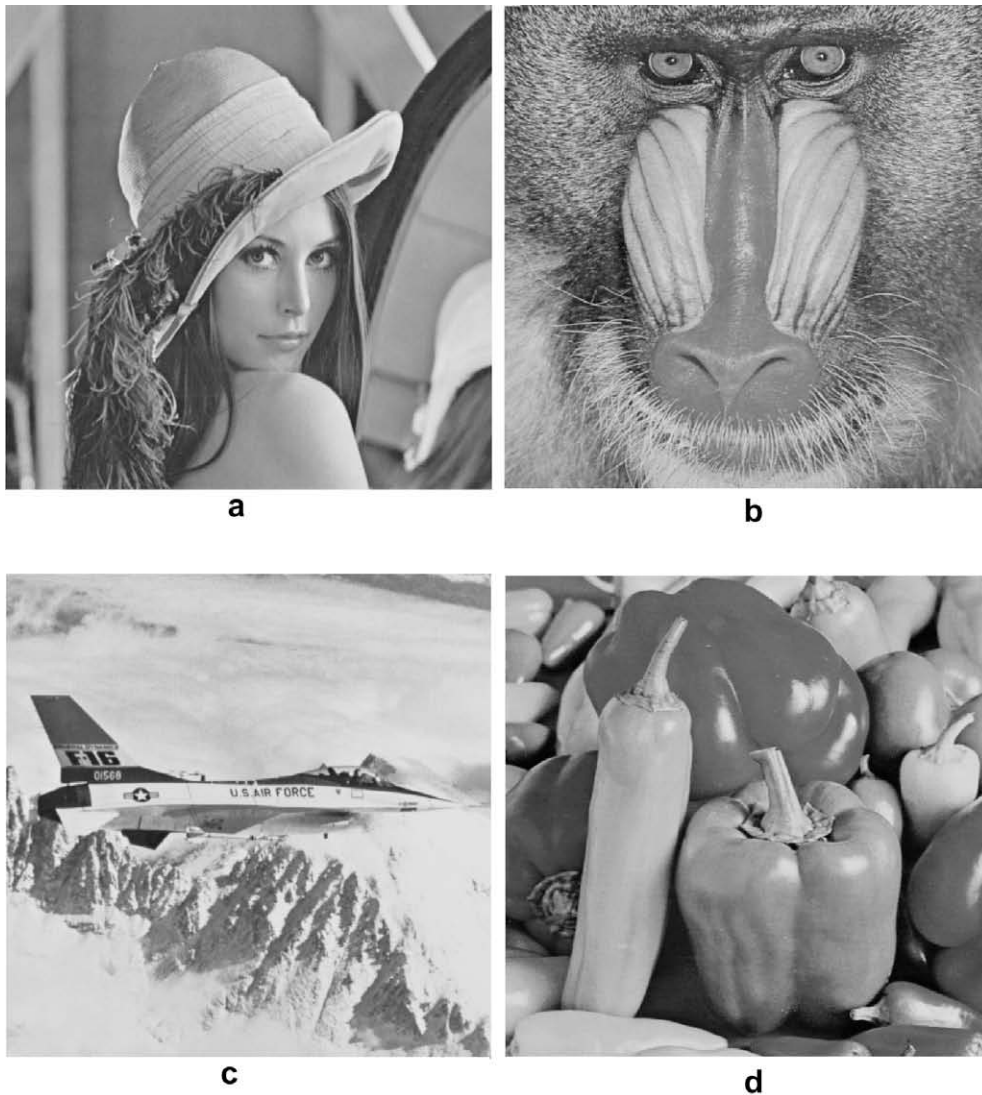


Fig. 1. Flowchart of the proposed kick-out method.



Abstract—The conception is of great interest in various or transmission of images. There has been a tendency to combine techniques in order to obtain good

In this paper, we propose an approach to image coding, based on wavelet transformations. The main contribution is that i) it relies on the assumption that the image can be efficiently exploited through a wavelet basis, and ii) it approximates the image using a

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Fig. 2. Five testing images: (a) Lena, (b) baboon, (c) F-16, (d) peppers and (e) text.

$$\|\widehat{R}\|_1 = \sum_{i=1}^n \sum_{j=1}^n |\widehat{r}_{ij}| = \sum_{i=1}^n |\widehat{r}_i| \quad (12)$$

where \widehat{r}_i denotes the i th element of \widehat{R} . The normalized one-norm of the domain block D is denoted by $\|\widehat{D}\|_1$. For convenience, “block” and “vector” are used interchangeably throughout this paper.

Before presenting our proposed encoding method, a new inequality is given below.

Theorem 1. Given a range block R and a domain block D , if $\|\widehat{R}\|_1 \geq \|\widehat{D}\|_1$, it yields to

$$E(R, D) \geq \frac{\|R - \bar{r}I\|^2}{N} \|\widehat{R}\|_1 - \|\widehat{D}\|_1^2 \quad (13)$$

where $\frac{\|R - \bar{r}I\|^2}{N}$ is the variance of R .

Proof. By (3), it is known $o = \bar{r} - \bar{s}d$. Let $A = R - \bar{r}I$ and $B = D - \bar{d}I$, then it yields to

$$\begin{aligned} E(R, D) &= \|R - (sD + oI)\|^2 = \|R - (sD + \bar{r}I - \bar{s}dI)\|^2 \\ &= \|R - \bar{r}I - s(D - \bar{d}I)\|^2 = \|A - sB\|^2. \end{aligned}$$

Denoting A and B as vectors, we have $A = \{a_1, \dots, a_N\}$ and $B = \{b_1, \dots, b_N\}$. By Cauchy inequality, it yields to

$$\begin{aligned} N \cdot E(R, D) &= \sum_{i=1}^N |a_i - sb_i|^2 \cdot \sum_{i=1}^N 1^2 \geq \left(\sum_{i=1}^N |a_i - sb_i| \right)^2 \\ &\geq \left(\sum_{i=1}^N \|a_i\| - \|sb_i\| \right)^2 \geq \left| \sum_{i=1}^N (\|a_i\| - \|s\| \|b_i\|) \right|^2. \end{aligned}$$

Due to

$$|s| = \frac{|\langle R - \bar{r}I, D - \bar{d}I \rangle|}{\|D - \bar{d}I\|^2} = \frac{|\langle A, B \rangle|}{\|B\|^2} \leq \frac{\|A\| \|B\|}{\|B\|^2}$$

and $\|\widehat{R}\|_1 \geq \|\widehat{D}\|_1$, we have

$$\begin{aligned} N \cdot E(R, D) &\geq \left| \sum_{i=1}^N \left(|a_i| - \frac{\|A\| \|B\|}{\|B\|^2} |b_i| \right) \right|^2 = \|A\|^2 \left| \sum_{i=1}^N \left(\frac{|a_i|}{\|A\|} - \frac{|b_i|}{\|B\|} \right) \right|^2 \\ &= \|A\|^2 \|\widehat{R}\|_1 - \|\widehat{D}\|_1^2. \end{aligned}$$

Equivalently, it yields to

$$E(R, D) \geq \frac{\|A\|^2}{N} \|\widehat{R}\|_1 - \|\widehat{D}\|_1^2 = \frac{\|R - \bar{r}I\|^2}{N} \|\widehat{R}\|_1 - \|\widehat{D}\|_1^2.$$

We have completed the proof in a straightforward way. \square

Let $E_{L1}(R, D) = \frac{\|R - \bar{r}I\|^2}{N} \|\widehat{R}\|_1 - \|\widehat{D}\|_1^2$. For R , assume that D_{min} is the current minimum error. Clearly, for any domain block D , if $\|\widehat{R}\|_1 \geq \|\widehat{D}\|_1$ and the inequality

$$E_{L1}(R, D) \geq D_{min} \quad (14)$$

is held, Theorem 1 implies $E(R, D) \geq D_{min}$. Therefore, the domain block D will not be the best matched block for R , i.e. it can be discarded immediately. This guarantees that the discarded domain

block D is indeed useless and does not affect the final decoded image quality, so our proposed kick-out method still keeps the same decoded image quality as in the full search method.

Initially, we compute $\|\widehat{D}_i\|_1$ of each domain block D_i for $1 \leq i \leq N_D$. Then, these N_D domain blocks are sorted according to the normalized one-norms in an increasing order. For each range block R , our proposed kick-out based fractal encoding method is listed below:

Step 1: For $\|\widehat{R}\|_1$, use binary search to find the domain block D_p such that $\|\widehat{R}\|_1 - \|\widehat{D}_p\|_1$ is minimal. For $1 \leq k \leq 8$, the eight errors $E(R, D_{p,k})$'s for the eight isometry transformations are computed. The current minimum error D_{min} is set to $\min_k \{E(R, D_{p,k})\}$. We have that D_p is the current best matched domain block of R . Set $q = 1$.

Step 2: If $p - q < 1$ or the domain block D_{p-q} has been discarded, go to Step 3.

Step 2.1: By (14), compute $E_{L1}(R, D_{p-q})$. If $E_{L1}(R, D_{p-q}) \geq D_{min}$, remove all the domain blocks from D_{p-q} to D_1 and go to Step 3.

Step 2.2: Compute the error $E(R, D_{p-q,k})$ for $1 \leq k \leq 8$. If $\min_k \{E(R, D_{p-q,k})\} < D_{min}$, D_{min} is set to $\min_k \{E(R, D_{p-q,k})\}$ and D_{p-q} is the current best matched domain block of R .

Step 3: If $p + q > N_D$, go to Step 4.

Step 3.1: Compute the error $E(R, D_{p+q,k})$ for $1 \leq k \leq 8$. If $\min_k \{E(R, D_{p+q,k})\} < D_{min}$, D_{min} is set to $\min_k \{E(R, D_{p+q,k})\}$ and D_{p+q} is the current best matched domain block of R .

Step 4: Set $q = q + 1$. If $(p - q < 1$ or D_{p-q} has been removed) and $(p + q > N_D)$, stop the search process. Otherwise, go to Step 2.

In Step 2.1, if the domain block D_i with $i < p$ satisfies the condition in (14), D_i and D_j with $j < i$ will not be possible best matched domain blocks, then they can be discarded since

$$\|\widehat{R}\|_1 - \|\widehat{D}_j\|_1 \geq \|\widehat{R}\|_1 - \|\widehat{D}_i\|_1 \quad \text{for all } j < i. \quad (15)$$

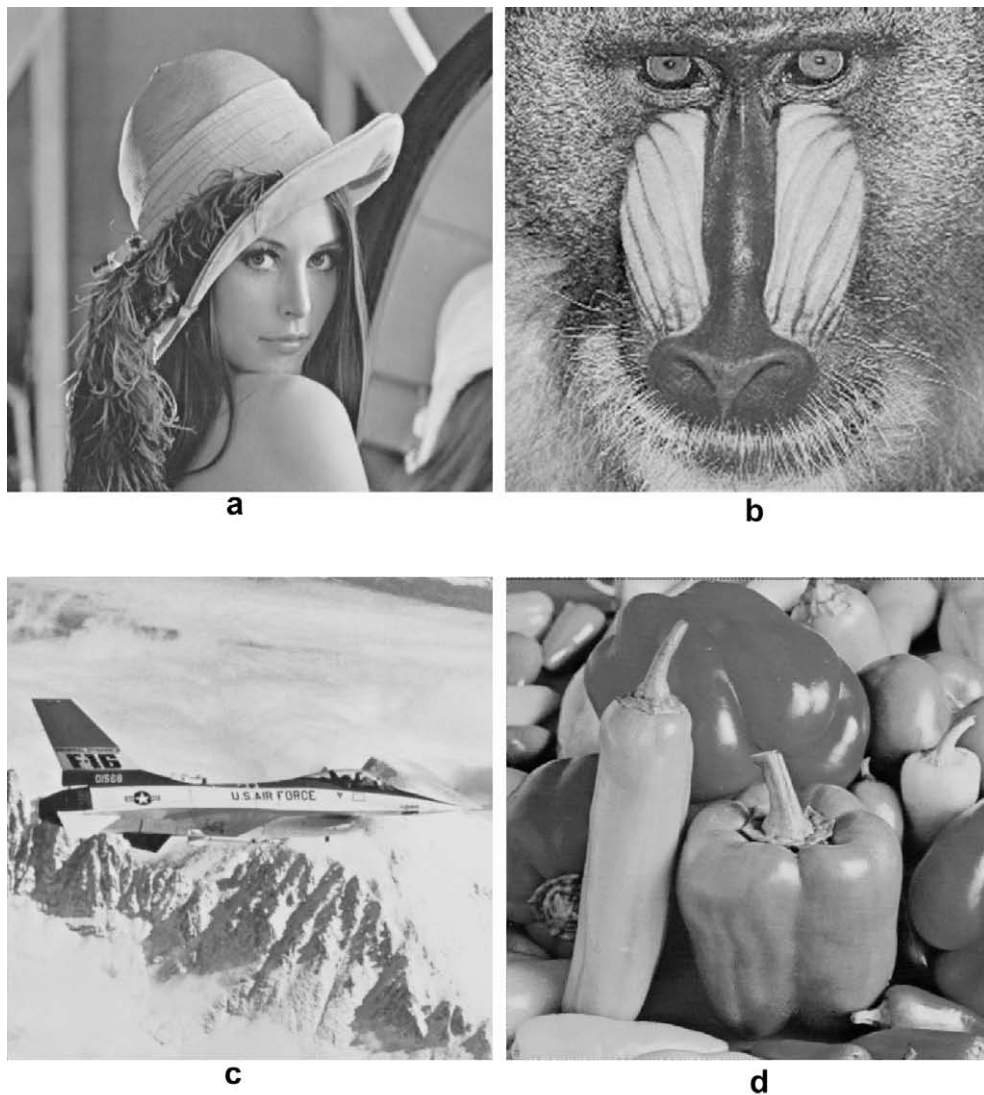
The flowchart of our proposed kick-out method for finding the best matched block is depicted in Fig. 1.

4. Experimental results

Five popular 512×512 images, Lena image, Baboon image, F16 image, Peppers image, and text image, as shown in Fig. 2, are used to compare the performance in terms of execution time and decoded image quality among the seven concerned fractal image encoding methods. The seven concerned methods are the full search method, Truong et al.'s DCT inner product method, Lai et al.'s method, our proposed method called “Ours”, the method by combining Ours and Truong et al.'s method called “Ours + Truong”, the method by combining Ours and Lai et al.'s method called “Ours + Lai”, and combining our proposed method and both methods called “Ours + Truong + Lai”.

Table 1
Execution time performance comparison for 4×4 range block.

	Lena	Baboon	F-16	Peppers	Text	Average	Improvement ratio (%)
Full search	255	256	257	252	279	259.8	0
Truong et al.'s method	165	159	161	165	171	164.2	37
Lai et al.'s method	199	202	192	191	189	194.6	25
Ours	201	213	200	209	123	189.2	27
Ours + Truong	124	128	122	121	70	113	57
Ours + Lai	131	137	134	135	116	130.6	50
Ours + Truong + Lai	79	82	82	81	67	78.2	70



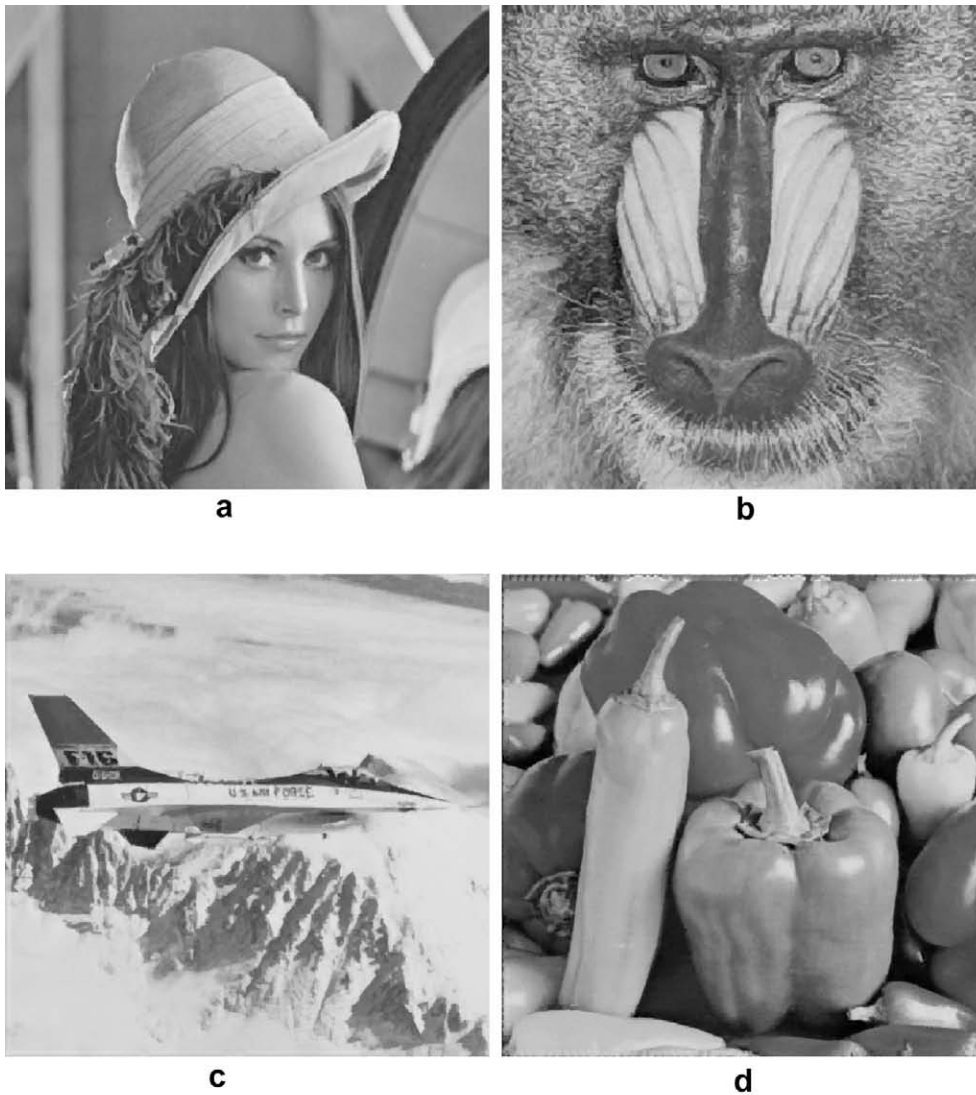
Abstract—The conception is of great interest in various or transmission of images. There has been a tendency to combine techniques in order to obtain good results. In this paper, we propose a new approach to image coding, based on wavelet transformations. The main contribution is that i) it relies on the assumption that the wavelet basis can be efficiently exploited through a sparse representation, and ii) it approximates the original image using a limited number of wavelet coefficients.

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Fig. 3. Five decoded images for range block 4×4 : (a) Lena, (b) baboon, (c) F-16, (d) peppers and (e) text.

All the concerned methods have been coded in Borland C++ Builder 6 on the 3.0 GHz PC with 1 GB RAM. The execution time required in each method is measured by seconds. The decoded

image qualities are measured by peak signal-to-noise ratio (PSNR) and the normalized correlation coefficient (NCC). The PSNR is defined by



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Fig. 4. Five decoded images for range block 8×8 : (a) Lena, (b) baboon, (c) F-16, (d) peppers and (e) text.

Table 2
Execution time performance comparison for 8 × 8 range block.

	Lena	Baboon	F-16	Peppers	Text	Average	Improvement ratio (%)
Full search	211	214	211	211	234	216.2	0
Truong et al.'s method	124	121	118	120	118	120.2	44
Lai et al.'s method	176	178	169	174	175	174.4	19
Ours	176	191	178	172	153	179.25	17
Ours + Truong	96	101	99	95	87	97.75	55
Ours + Lai	128	131	122	138	120	127.8	41
Ours + Truong + Lai	69	72	63	69	59	66.4	69

Table 3
Decoded image quality of the testing images for 4 × 4 and 8 × 8 range block.

		Lena	Baboon	F-16	Peppers	Text
4 × 4	PSNR	37.56	30.30	36.44	36.51	29.32
	NCC	0.99968	0.99838	0.99979	0.99949	0.99926
8 × 8	PSNR	31.34	23.32	29.22	25.86	18.32
	NCC	0.99865	0.99789	0.99886	0.99493	0.99059

$$PSNR = 10 \log_{10} \frac{255 * 255 * M_f * N_f}{\sum_{x=1}^{M_f} \sum_{y=1}^{N_f} (f(x, y) - f'(x, y))^2} \quad (16)$$

where $M_f * N_f$ denotes the image size, $f(x, y)$ is the original image pixel value at position (x, y) , and $f'(x, y)$ is the corresponding decoded image pixel value. The NCC is defined by

$$NCC = \frac{\sum_{x=1}^{M_f} \sum_{y=1}^{N_f} f(x, y) \cdot f'(x, y)}{\sqrt{\sum_{x=1}^{M_f} \sum_{y=1}^{N_f} f(x, y)^2} \cdot \sqrt{\sum_{x=1}^{M_f} \sum_{y=1}^{N_f} f'(x, y)^2}} \quad (17)$$

Tables 1 and 2 demonstrate the execution-time performance comparison for the seven concerned encoding methods. The decoded five images are shown in Fig. 3 for 4 × 4 range block. Fig. 4 illustrates the decoded five images for 8 × 8 range block. The domain blocks are obtained by sliding a window in the same input image with the incremental step of eight pixels horizontally or vertically. In our proposed method, the time required for computing the normalized one-norm of each domain block and the time required for sorting these normalized one-norms are included in the encoding time to assure fair comparison. From Tables 1–2, in average, the encoding time improvement ratios of our proposed method, Truong et al.'s method, and Lai et al.'s method over the full search method are 22%, 40.5%, and 22%, respectively. The execution time improvement ratios of Ours + Truong, Ours + Lai, and Ours + Truong + Lai are 56%, 45.5%, 69.5%, respectively. For the same five test images, the above seven concerned methods have the same decoded image quality and it is shown in Table 3.

5. Conclusions

In this paper, we have presented the proposed improved fractal image encoding method. The main contribution of this paper is to present a new inequality which can be used to eliminate the impossible domain blocks for the current range block efficiently. The fractal image encoding process can therefore be speeded up efficiently. Based on five typical testing images, for 4 × 4 and 8 × 8 range blocks, our proposed kick-out method has 22% execution time improvement ratio in average. Further, we investigate the combined version of our proposed method and Lai et al.'s method, Truong et al.'s method, or both methods. Experimental results confirm the computational merits. All the concerned methods have the same decoded image quality as in Jacquin's full search method mentioned in the introduction.

From experimental results, we find that the larger the range block is, the worse the decoded image quality is; the higher the frequency of the image contents is, the worse the decoded image quality is. It is a research issue to integrate the fractal encoding method with the spatial approach and the frequency approach [22] to alleviate this side-effect. Further, how to include the color information into the fractal image coding is another interesting research topic.

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