Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER
Chaos, Solitons and Fractals 29 (2006) 215–222
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

Novel prediction- and subblock-based algorithm for fractal image compression ☆

Kuo-Liang Chung *, Chung-Hsiang Hsu

Department of Computer Science and Information Engineering, National Taiwan University of Science and Technology, No. 43, Section 4, Keelung Road, Taipei 10672, Taiwan ROC

Accepted 16 August 2005

Abstract

Fractal encoding is the most consuming part in fractal image compression. In this paper, a novel two-phase prediction- and subblock-based fractal encoding algorithm is presented. Initially the original gray image is partitioned into a set of variable-size blocks according to the S-tree- and interpolation-based decomposition principle. In the first phase, each current block of variable-size range block tries to find the best matched domain block based on the proposed prediction-based search strategy which utilizes the relevant neighboring variable-size domain blocks. The first phase leads to a significant computation-saving effect. If the domain block found within the predicted search space is unacceptable, in the second phase, a subblock strategy is employed to partition the current variable-size range block into smaller blocks to improve the image quality. Experimental results show that our proposed prediction- and subblock-based fractal encoding algorithm outperforms the conventional full search algorithm and the recently published spatial-correlation-based algorithm by Truong et al. in terms of encoding time and image quality. In addition, the performance comparison among our proposed algorithm and the other two algorithms, the no search-based algorithm and the quadtree-based algorithm, are also investigated.

1. Introduction

Fractal image compression was first invented by Barnsley [1] according to the contractive mapping fixed-point theorem. In his proposed iterated function system (IFS), the contacted transform consisting of a sequence of affine transformations is applied to the entire image. Later Jacquin [12,13,24] proposed a partitioned IFS (PIFS) associated with a block-based automatic encoding algorithm where those affine transformations are applied to partitioned blocks. Fractal image compression can be used in many applications such as image retrieval [16,17], watermark [18,25], multimedia encyclopedia [2], and hybrid coding methods [5,11,15,22].

The bottleneck in the PIFS fractal coding scheme is time-consuming in the encoding process. In order to alleviate this serious encoding time problem, several efficient fractal encoding algorithms have been developed. These

☆ Supported by National Council of Science of ROC under contract NSC93-2213-E-011-023.
* Corresponding author.
E-mail address: klchung@cs.ntust.edu.tw (K.-L. Chung).

0960-0779/$ - see front matter
doi:10.1016/j.chaos.2005.08.023

developed encoding algorithms include partitioned-based approach [6–9,19], domain pool selection approach [3,9], and search strategy-based approach [10,14]. Recently, Furao and Hasegawa [10] presented a no search fractal encoding method and experimental results showed the speedup of the encoding time when compared to the previous method by Tong and Wong [21], but having little image quality degradation. Note that Tong and Wong's encoding method has better image quality and encoding time performance when compared with Saupe's fractal coding method [20]. Currently, Truong et al. [23] presented an efficient spatial-correlation-based algorithm for fractal encoding and their proposed algorithm had a significant improvement when compared with the baseline algorithm, i.e. the full search algorithm.

In this paper, a two-phase prediction- and subblock-based fractal encoding algorithm is presented. Initially the original gray image is partitioned into a set of variable-size blocks according to the S-tree- and interpolation-based decomposition principle. In the first phase, each current block of variable-size range block tries to find the best matched domain block based on the proposed prediction-based search strategy which utilizes the relevant neighboring variable-size domain blocks. The first phase leads to a significant computation-saving effect. If the domain block found within the predicted search space is unacceptable, in the second phase, a subblock strategy is employed to partition the current variable-size range block into smaller blocks to improve the image quality. Experimental results show that our proposed prediction- and subblock-based fractal encoding algorithm outperforms the conventional full search algorithm and the recently published spatial-correlation-based algorithm by Truong et al. [22] in terms of encoding time and image quality. In addition, the performance comparison among our proposed algorithm and the other two algorithms, the no search-based algorithm and the quadtree-based algorithm, are also investigated.

The remainder of this paper is organized as follows. In Section 2, three relevant past works are surveyed. In Section 3, our proposed prediction- and subblock-based encoding algorithm is presented. In Section 4, some experimental results are demonstrated. Conclusions are addressed in Section 5.

2. The three past works

Before introducing the past three relevant fractal encoding algorithms, namely the quadtree-based approach, the no search-based approach, and the spatial-correlation-based approach, let us introduce some preliminary background of fractal image compression.

Suppose the original image is of size $m \times m$. In the conventional PIFS fractal encoding scheme [12], the original image is partitioned into a set of nonoverlapping range blocks, which constitute the range pool, and a set of overlapping domain blocks, which constitute the domain pool. Usually each range block is of size 8×8 and each domain block is of size 16×16. For simplicity, let all the partitioned range blocks be denoted by the set $R = \{R_i, 1 \leqslant i \leqslant N_R\}$ where N_R ($=m^2/64$) denotes the number of range blocks; let the partitioned domain blocks be denoted by the set $D = \{D_i, 1 \leqslant i \leqslant N_D\}$ where N_D denotes the number of domain blocks in the domain pool; let the shrunk domain block be denoted by $\widehat{D} = \{\widehat{D}_{ik}, 1 \leqslant i \leqslant N_D, 1 \leqslant k \leqslant 8\}$. To match the size of range block, $\widehat{D}_{ik}$ is obtained by averaging four pixels to one pixel for each domain block under the kth isometric operation.

Considering all the four isometric rotations and four affine transformations, the shrunk domain pool is defined to be the set $\{\widehat{D}_{i1}, \widehat{D}_{i2}, \ldots, \widehat{D}_{i8}, 1 \leqslant i \leqslant N_D\}$. When the scaling factor s and the offset factor o are set to

$$s = \frac{\langle R, \widehat{D}\rangle - \frac{1}{k}\langle R, 1\rangle\langle \widehat{D}, 1\rangle}{\|\widehat{D}\|^2 - \frac{1}{k}\langle \widehat{D}, 1\rangle^2} \text{ and } o = \frac{1}{k}(\langle R, 1\rangle - s\langle \widehat{D}, 1\rangle) \tag{1}$$

the collage error $E_c(R, \widehat{D}) = \|R - (s\widehat{D} + o1)\|^2$ can be minimized. The coefficient s should be confined in the interval $[-1, 1]$ to ensure the convergence in decoding.

In [9], Fisher presented a quadtree-based fractal encoding algorithm. In Fisher's algorithm, if the collage error between the range block and the best matched domain block is larger than the threshold, the current range block is further decomposed into four quadrants and we find the best matched domain block for each quadrant in this recursive quadtree decomposition way; otherwise, the best matched domain block can be determined.

In [10], Furao and Hasegawa presented a no search-based fractal encoding algorithm. To speed up the encoding process, they utilize the correlation between the current range block and the domain block centered on the current range block. In their no search-based encoding scheme, the best matched domain block of current range block is depicted in Fig. 1 where the current range block is of size 16×16 and the specified domain block is of size 32×32. If the collage error between the range block and the specified domain block is greater than the threshold, the current range block is decomposed by using the quadtree decomposition principle. For each quadrant, i.e. a variable-size subrange-block, the quadtree–based fractal encoding approach [9] is employed to find the best matched domain block.

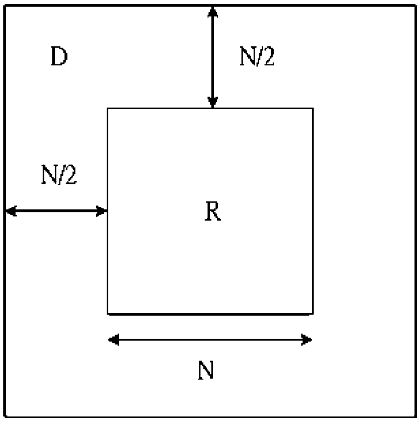

Fig. 1. The relationship between the current range block and the specified domain block in no search-based approach.

According to the spatial correlation between the current range block and the four neighboring range blocks, Truong et al. [23] presented an efficient fractal encoding algorithm. As shown in Fig. 2, let the current range block be denoted by $R_c \in R$ and the four neighboring range blocks be denoted by R_{nw}, R_n, R_{ne}, and R_w, respectively, to the northwest direction, north direction, northeast direction, and west direction. Suppose the best matched domain blocks of R_{nw}, R_n, R_{ne}, and R_w are denoted by D_{nw}, D_n, D_{ne}, and D_w, respectively, in the domain pool.

In [23], for the current range block R_c, 16 domain blocks should be examined. Among these 16 examined domain blocks, each set consisting of four consecutive domain blocks is considered for each concerning direction. For example, for west direction of current range block R_c, four domain blocks, say $D^0_w, D^1_w\ (= D_w), D^2_w$, and D^3_w, should be examined. Truong et al.'s search strategy first try to find the best matched shrunk domain block of R_c under one specific affine transform among the 16 concerned domain blocks. If the collage error between the corresponding best matched shrunk domain block and the current range block R_c is less than the threshold, the fractal encoder thus records the position of the best matched domain block, the value of s (see Eq. (1)), the value of o (see Eq. (1)), and the index of the corresponding affine transform. Otherwise, the full search strategy is employed to find the best matched domain block in the domain pool.

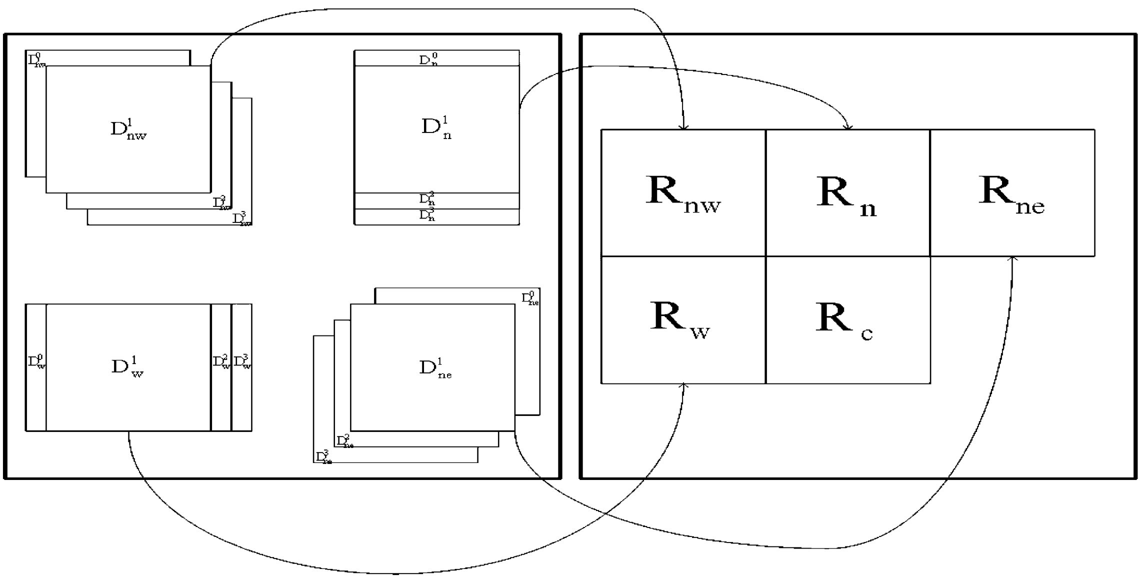

Fig. 2. The searching space of the current range block in spatial-correlation-based approach.

3. The proposed fractal encoding method

In this section, our proposed prediction- and subblock-based fractal encoding algorithm is presented.

In order to distinguish our proposed subblock approach from Fisher's quadtree approach, we first point out the shortcoming in computing time of Fisher's approach. In Fisher's fractal encoding algorithm [10], if the collage error between the current range block and the best matched domain block is larger than the threshold, the current range block is further decomposed into four quadrants and we want to find the best matched domain block for each quadrant by using this recursive quadtree decomposition way. The main drawback in Fisher's quadtree approach is that before decomposing one quadrant into smaller ones, we have examined the whole larger quadrants whether are well suited or not. For one final decomposed quadrant in the original current range block, checking these redundant larger quadrants spends enormous computing time. In order to overcome this time-consuming shortcoming in Fisher's approach, we propose a subblock approach to partition the whole original image into a set of variable-size blocks in advance according to the quadtree- and interpolation-based technique [4]. These partitioned variable-size blocks constitute the range pool. During partitioning the original image into a set of blocks, a block is called a homogeneous block if the estimated greylevel of each pixel in the block is in some vicinity of its real greylevel. Suppose the coordinates of the four corners in a block are denoted by (x_1, y_1), (x_2, y_1), (x_1, y_2), and (x_2, y_2) with greylevels g_1, g_2, g_3, and g_4, respectively. The estimated greylevel of the pixel at (x, y) in the block is calculated by

$$\begin{aligned} g_{\text{est}}(x,y) &= g_5 + \frac{g_6 - g_5}{y_2 - y_1}(y - y_1), \quad \text{where} \\ g_5 &= g_1 + \frac{g_2 - g_1}{x_2 - x_1}(x - x_1) \quad \text{and} \\ g_6 &= g_3 + \frac{g_4 - g_3}{x_2 - x_1}(x - x_1) \end{aligned} \tag{2}$$

Given a specified error tolerance ε, if the following image quality condition holds:

$$|g(x,y) - g_{\text{est}}(x,y)| \leqslant \varepsilon \tag{3}$$

for all the estimated pixels at position (x, y) in the block, $x_1 \leqslant x \leqslant x_2$ and $y_1 \leqslant y \leqslant y_2$, then the block is called a homogeneous block. Fig. 3 illustrates one example containing a set of homogeneous blocks and the corresponding quadtree representation.

Besides the above proposed subblock approach, a novel prediction-based strategy is further presented to determine the suitable search window. The determined search window can reduce the search space significantly. The search window is determined based on the mean of the relevant motion vectors of the neighboring range subblocks of the current range block. As shown in Fig. 4, it is known that the current range block is R_c and its neighboring range subblocks are denoted by R_{nw}, R_n, R_{ne}, and R_w. In fact, each neighboring range subblock might be a subblock of the corresponding range block. Assume the four corresponding matched variable-size domain blocks of R_{nw}, R_n, R_{ne}, and R_w are denoted by D_{nw}, D_n, D_{ne}, and D_w. Utilizing the mean value of the motion vectors of four matched domain blocks as center, then a search window with size 33×33 is generated. If the best matched domain block of the current range block R_c is found below threshold within the generated search window, the position of the best matched domain block, the value of s, the value of o, and the index of the corresponding affine transform are recorded. Otherwise, if the domain block found within the search window is unacceptable, a recursive quadtree-based partition strategy is used to partition the current variable-size range block into smaller blocks to find the best domain block until the collage error is less than the specified threshold.

According to our proposed prediction- and subblock-based approach, the proposed fractal encoding algorithm is listed below:

Step 1: Partition the original image into a set of homogeneous blocks using the quadtree- and interpolation-based decomposition principle [4]. These homogeneous blocks constitute the range pool.

Step 2: For each variable-size range block R_i, we first determine the search window using the mean value of motion vectors of the relevant neighboring domain blocks. Compute the collage error $E_c(R_i, D) = \|R_i - (sD + o1)\|^2$ between R_i and each candidate domain block within the search window. If all the candidate domain blocks fail, go to Step 3; otherwise go to Step 4.

Step 3: The range block R_i is decomposed into four quadrants based on the quadtree decomposition principle. Go to Step 2.

Step 4: Record s_i, o_i, location of the best matched domain block, and the index of the used affine transformation as the encoding data.

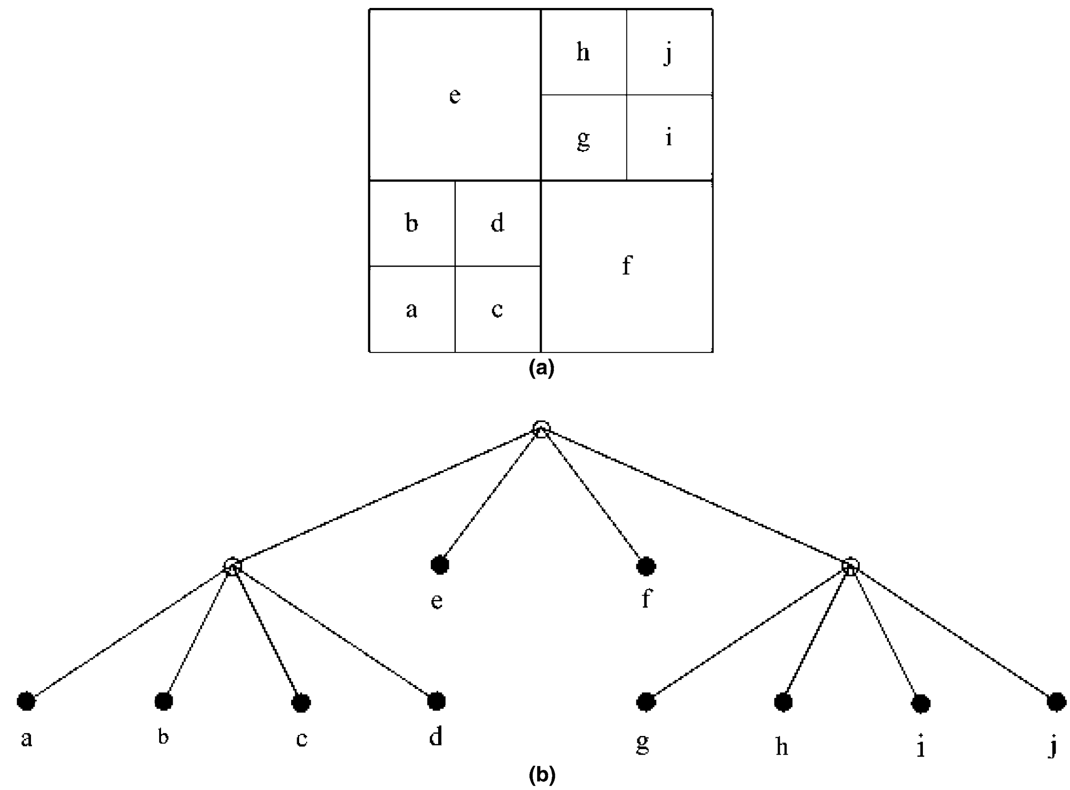

Fig. 3. One example. (a) Homogeneous blocks and (b) the quadtree representation.

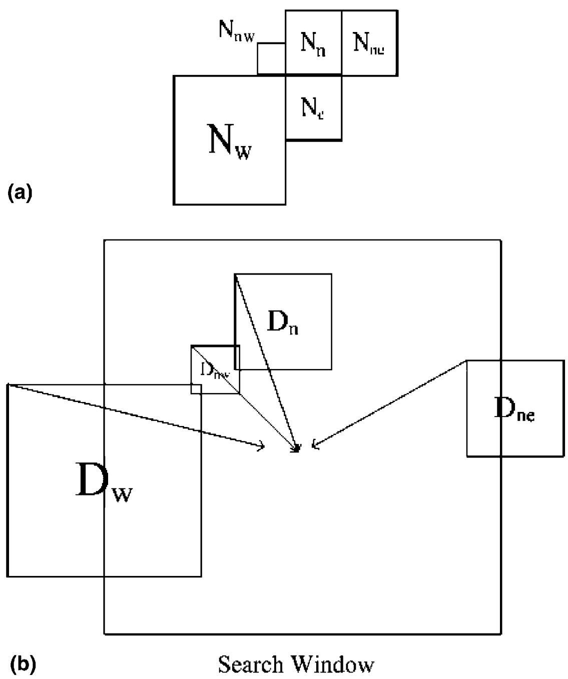

Fig. 4. The search window determination. (a) The current variable-size range block and its neighboring range blocks and (b) the determined search window.

4. Experimental results

Four real 256×256 images, Lenna, Pepper, Baboon, and F16 as shown in Fig. 5, are used to evaluate the relative performance in terms of execution time and image quality among the concerned five fractal image algorithms. All the above algorithms have been coded in Borland C++ Builder 6 on the personal computer with Pentium 4 3.0 GHz and 512 MB RAM.

The execution time required in each fractal encoding algorithm is measured by seconds. The performance of the decoded image quality is measured by PSNR (peak signal-to-noise ratio) and PSNR is defined by

$$\mathrm{PSNR} = 10\log_{10}\frac{255 * M * N}{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[I(x,y) - I'(x,y)]^2} \tag{4}$$

where $M \times N$ denotes the image size; $I(x,y)$ is the original image pixel value at location (x,y) and $I'(x,y)$ is the decoded image pixel value. In the five concerned algorithms, the threshold value of the collage error is set to 1500. The size of each range block in the full search algorithm and in the spatial-correlation-based algorithm is of 8×8. In our proposed algorithm, the value of ε is selected to be 4 for four testing images. In fact, for different image, the value of ε can be determined by experimental evidence. The size of each range block in the quadtree-based and in the no search-based algorithm is of 16×16.

Table 1 demonstrates the performance comparison among the five fractal encoding algorithms. From Table 1, our proposed algorithm is faster than the full search algorithm, the quadtree-based algorithm, and the spatial-correlation-based algorithm, but is slower than the no search-based algorithm. Our proposed algorithm has better image quality when compared to the full search algorithm [1], the spatial-correlation-based algorithm [23], and the no search-based algorithm [10], but has worse image quality when compared to the quadtree-based algorithm. Based on the four testing images, the execution-time improvement ratio of the proposed algorithm over the full search algorithm is 88% in average and the PSNR improvement ratio is 10% in average. The execution-time improvement ratio of the proposed algo-

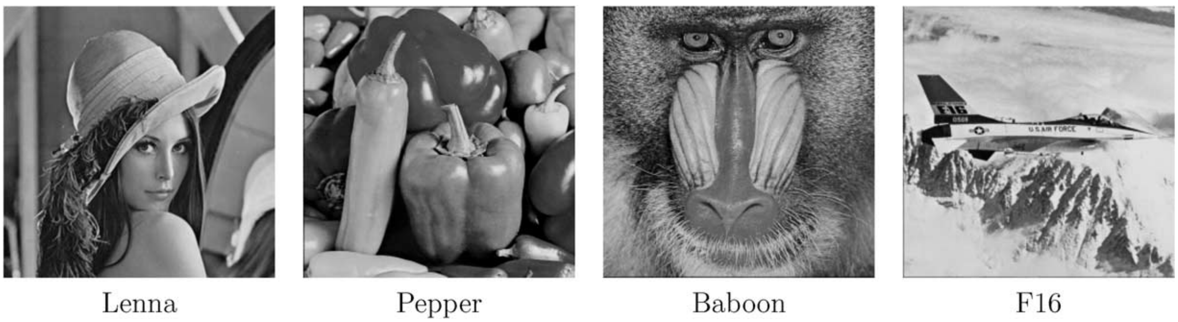

Fig. 5. Four tested images.

Table 1
Performance comparison among the five concerned algorithms

		Lenna	Pepper	Baboon	F16	Average
Full search [12]	PSNR	28.6	29.5	23.4	26.7	27.1
	Time	488.4	482.3	485.9	482.4	484.8
Quadtree [9]	PSNR	36.1	36.6	28.8	36.2	34.4
	Time	1630.0	1628.1	2390.3	1670.7	1829.8
No search [10]	PSNR	29.1	30.2	21.6	26.6	26.9
	Time	0.1	0.1	0.1	0.1	0.1
Spatial correlation [23]	PSNR	28.5	29.0	23.4	26.6	26.9
	Time	260.5	246.2	430.4	253.4	297.6
Ours	PSNR	31.1	31.1	25.9	31.8	30.0
	Time	58.7	58.7	57.9	64.1	59.9

Lenna Pepper Baboon F16

Fig. 6. The decompressed images by using our proposed method.

rithm over the spatial-correlation-based algorithm is 80% in average and the PSNR improvement ratio is 12% in average. In summary, considering both the execution-time requirement and the image quality assessment, our proposed two-phase prediction- and subblock-based fractal encoding algorithm is a good choice when compared to the previous four algorithms [12,9,10,23]. Fig. 6 illustrates the four decompressed images by using our proposed fractal image algorithm. When comparing Fig. 5 to Fig. 6, the four decompressed images by using our proposed fractal image algorithm are quite similar to the four original images from human visual system.

5. Conclusion

We have presented a novel prediction- and subblock-based fractal encoding algorithm. Initially the original gray image is partitioned into a set of variable-size blocks according to the S-tree- and interpolation-based decomposition principle. In the first phase, each current block of variable-size range block tries to find the best matched domain block based on the proposed prediction-based search scheme. If the domain block found within the predicted search space is unacceptable, in the second phase, a subblock-decomposition scheme is used to partition the current variable-size range block into smaller blocks to improve the image quality. Based on the four real images, experimental results show the proposed algorithm is a good choice to replace with the previous four algorithms [12,9,10,23].

References

[1] Barnsley MF. Fractals everywhere. San Diego: Academic; 1993.
[2] Barnsley MF. Fractal image compression. In: Blackledge JM, editor. Image processing: mathematical methods and applications. Oxford: Clarendon; 1997. p. 183–210.
[3] Barthel KU, Voye T. Adaptive fractal image coding in the frequency domain. In: Proceedings of international workshop on image processing, June 1994, p. 33–8.
[4] Chung KL, Wu JG. Improved image compression using s-tree and shading approach. IEEE Trans Commun 2000;5:748–51.
[5] Davis GM. A wavelet-based analysis of fractal image compression. IEEE Trans Image Process 1998;2:141–54.
[6] Davoine F, Chassery JM. Adaptive Delaunay triangulation for attractor image coding. In: The 12th international conference on pattern recognition, October 1994, p. 801–3.
[7] Davoine F, Svensson J, Chassery JM. A mixed triangular and quadrilateral partition for fractal image coding. In: Proceedings of international conference on image processing, vol. 3, October 1995, p. 284–7.
[8] Davoine F, Antonini M, Chassery JM, Barlaud M. Fractal image compression based on Delaunay triangulation and vector quantization. IEEE Trans Image Process 1996;2:338–46.
[9] Fisher Y. Fractal image compression, theory and application. New York: Springer-Verlag; 1994.
[10] Furao S, Hasegawa O. A fast no search fractal image coding method. Signal Process Image Commun 2004;5:393–404.
[11] Fu P, Tang X, Zhu Y, Wu X. A new fractal block coding scheme based on classification in the wavelet domain. In: Proceedings of the IEEE international vehicle electronics conference (IVEC '99), vol. 1, 1999. p. 315–8.
[12] Jacquin AE. Image coding based on a fractal theory of iterated contractive image transform. IEEE Trans Image Process 1992;1:18–30.
[13] Jacquin AE. Fractal image coding: a review. In: Proceedings of the IEEE, vol. 10, October 1993, p. 1451–65.
[14] Kominek J. Algorithm for fast fractal image compression. In: Proceedings of SPIE, vol. 2419, February 1995, p. 296–305.
[15] Li J, Kuo CJ. Image compression with a hybrid wavelet-fractal coder. IEEE Trans Image Process 1999;6:868–74.

[16] Nappi M, Polese G, Tortora G. FIRST: fractal indexing and retrieval sysTem for image databases. Image Vis Comput 1998;14:1019–31.
[17] Distasi R, Nappi M, Tucci M. FIRE: fractal indexing with robust extensions for image databases. IEEE Trans Image Process 2003;3:373–84.
[18] Pi MH, Li CH. A novel fractal watermarking technique. In: Proceedings of IEEE international conference on acoustics, speech, and signal processing (ICASSP '04), vol. 5, May 2004, p. 369–72.
[19] Saupe D, Jacob S. Variance-based quadtrees in fractal image compression. Electron Lett 1997;1:46–8.
[20] Saupe D, Hartenstein H. Lossless acceleration of fractal image compression by fast convolution. In: Proceedings of IEEE international conference on image processing, vol. 1, September 1996, p. 185–8.
[21] Tong CS, Wong M. Adaptive approximate nearest neighbor search for fractal image compression. IEEE Trans Image Process 2002;6:605–14.
[22] Truong TK, Jeng J, Reed IS, Lee PC, Li AQ. A fast encoding algorithm for fractal image compression using the DCT inner product. IEEE Trans Image Process 2000;4:529–35.
[23] Truong TK, Kung CM, Jeng JH, Hsieh ML. Fast fractal image compression using spatial correlation. Chaos, Solitons & Fractals 2004;3:1071–6.
[24] Wohlberg B, Gerhard DJ. A review of the fractal image coding literature. IEEE Trans Image Process 1999;12:1716–29.
[25] Yao Z. Fixed point in fractal image compression as watermarking. In: Proceedings of international conference on image processing, vol. 2, September 2003, p. 475–8.