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ScienceDirect
Applied Mathematics and Computation 188 (2007) 54–57
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

On SVD-based watermarking algorithm

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Abstract

This short communication presents two notes for singular value decomposition (SVD)-based watermarking scheme. The presented notes can increase the invisibility and capacity when embedding the watermark into U and V components of the SVD.

Keywords: Invisibility; Capacity; SVD; Watermark

1. Introduction

Given the matrix A which represents an input image, singular value decomposition (SVD) can be used to decompose A into $A = UDV^T$ where U and V are orthogonal matrices and D is a singular, diagonal matrix [3]. In applications, SVD technique has been applied to image compression [3], image hiding [2], and image watermarking [1,4].

Chung et al. [2] presented an SVD- and vector quantization-based image hiding algorithm for embedding the secret data into the D component of the SVD. Using a different way, Liu and Tan [4] presented an efficient SVD-based algorithm to modify the coefficients in D component for embedding watermark into the cover image. Chang et al. [1] presented a block-based watermarking algorithm by partitioning the image into several blocks and modifying the coefficients in U component for each block to achieve the watermarking effect.

In this short communication, two notes are presented to improve SVD-based watermarking algorithm. These two notes give a guideline to modify the coefficients in both U and V components in order to increase the invisibility and capacity when embedding the watermark into U and V components of the SVD.

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doi:10.1016/j.amc.2006.09.117

2. Our proposed notes

Suppose that the input image is represented by a matrix A. For convenience, assume that A is an $N \times N$ square matrix with rank r, $r \leqslant N$. The SVD of A can be represented by

$$A = UDV^{\mathrm{T}} = \begin{bmatrix} u_{1,1} & \cdots & u_{1,N} \\ u_{2,1} & \cdots & u_{2,N} \\ \vdots & \ddots & \vdots \\ u_{N,1} & \cdots & u_{N,N} \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \sigma_N \end{bmatrix} \begin{bmatrix} v_{1,1} & \cdots & v_{1,N} \\ v_{2,1} & \cdots & v_{2,N} \\ \vdots & \ddots & \vdots \\ v_{N,1} & \cdots & v_{N,N} \end{bmatrix}^{\mathrm{T}}, \tag{1}$$

where U and V are $N \times N$ orthogonal matrices and D is an N by N singular, diagonal matrix with diagonal entries σ_i's (singular values) satisfying $\sigma_1 \geqslant \sigma_2 \geqslant \cdots \geqslant \sigma_r > \sigma_{r+1} = \cdots = \sigma_N = 0$.

To explain our proposed two notes easily, assume that the input image A is a 4×4 matrix and the SVD of A is given by

$$A = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 \\ A_5 & A_6 & A_7 & A_8 \\ A_9 & A_{10} & A_{11} & A_{12} \\ A_{13} & A_{14} & A_{15} & A_{16} \end{bmatrix} = UDV^{\mathrm{T}}$$

$$= \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \begin{bmatrix} b_1 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & b_3 & 0 \\ 0 & 0 & 0 & b_4 \end{bmatrix} \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ c_5 & c_6 & c_7 & c_8 \\ c_9 & c_{10} & c_{11} & c_{12} \\ c_{13} & c_{14} & c_{15} & c_{16} \end{bmatrix}^{\mathrm{T}}.$$

After performing the matrix multiplications for UDV^{T}, the four pixels A_1, A_2, A_3, and A_4 are given by $b_1c_1a_1 + b_2c_2a_2 + b_3c_3a_3 + b_4c_4a_4$, $b_1c_5a_1 + b_2c_6a_2 + b_3c_7a_3 + b_4c_8a_4$, $b_1c_9a_1 + b_2c_{10}a_2 + b_3c_{11}a_3 + b_4 \; c_{12}a_4$, and $b_1c_{13}a_1 + b_2c_{14}a_2 + b_3c_{15}a_3 + b_4 \; c_{16}a_4$, respectively. Because the four coefficients $\{a_i| \; 1 \leqslant i \leqslant 4\}$ in the U component are involved in each of $\{A_i|1 \leqslant i \leqslant 4\}$, the distortion caused by modifying the coefficients of $\{a_i|1 \leqslant i \leqslant 4\}$ alters each of the four pixels $\{A_i|1 \leqslant i \leqslant 4\}$. For example, if we set the three coefficients $\{a_i|2 \leqslant i \leqslant 4\}$ equal to zero, A_1, A_2, A_3, and A_4 will be changed to $b_1c_1a_1$, $b_1c_5a_1$, $b_1 \; c_9a_1$, and $b_1c_{13}a_1$, respectively. In this case, A_i, $1 \leqslant i \leqslant 4$, changes from the sum of four terms to a signal term and this abrupt distortion may be detected immediately. The above analysis leads to the fact that modifying a row vector of the U component may cause abrupt distortion on only four pixels of A. This abrupt and uniform distortion may decrease the invisibility of the embedded watermark.

Distortion caused by modifying the coefficients in a row vector of the U component to be visible is that distortion is dispersed to four pixels of A only. A possible improvement can be achieved by modifying the coefficients in a column vector of the U component, which disperses distortion to twelve pixels of A. For example, if we set $\{a_i|i = 5, 9, 13\}$ equal to zero, each of $\{A_i|5 \leqslant i \leqslant 16\}$ is given by the sum of three terms instead of the sum of four terms, indicating a smooth change for each of the twelve pixels. Based on the above description, we suggest modifying the column vector instead of the row vector of U to perform watermarking.

Note 1: For the U component of the SVD, modifying the coefficients in column vector will cause less visible distortion than modifying the coefficients in row vector.

The analysis for Note 1 can also be applied on V^{T} which leads to the second note.

Note 2: For the V^{T} component of the SVD, modifying the coefficients in the row vector will cause less visible distortion than modifying the coefficients in column vector.

Notes 1 and 2 suggest that simultaneously modifying a column vector of the U component and a row vector of the V^{T} component can increase the invisibility and capacity of Chang et al.'s watermarking algorithm [1]. In [1], the image A is first partitioned into blocks evenly and the SVD is performed on each block. The watermark is embedded by changing the relation between the second and third coefficients in the first column of U component. If the embedded watermarking bit is *1*, the coefficients $u_{2,1}$ and $u_{3,1}$ must be modified to satisfy the condition $|u_{2,1}| - | \; u_{3,1}| \geqslant T$ where $|x|$ denotes the absolute value of x and T is the threshold; otherwise,

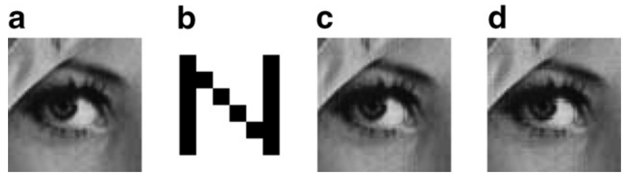

Fig. 1. An example of watermaring. (a) Original image. (b) Watermark. (c) Embedding (b) into U of (a). (d) Embedding (b) into both U and V of (a).

the condition $|u_{3,1}| - |u_{2,1}| > T$ must be held. Larger threshold T results in watermarked image with lower PSNR (Peak Signal–to–Noise Ratio) but makes the embedded watermark more robust. Chang et al.'s watermarking algorithm modifies a column vector of the U component to increase invisibility as suggested by Note 1. The capacity of the Chang et al.'s watermark algorithm can be increased by further modifying a row vector of the V^T component suggested by Note 2.

Fig. 1 shows some results of the above watermark embedding process. Fig. 1(a) is a 64×64 original image. Fig. 1(b) is the watermark to be embedded. Partition Fig. 1(a) into 8×8 blocks and then perform the above watermark embedding process with threshold $T = 0.002$. Fig. 1(c) and (d) shows the watermarked image by embedding the watermark into the U component only and both U and V components, respectively. The PSNRs of Fig. 1(c) and (d) are 43.52 and 42.69, respectively, indicating that the rules suggested by the proposed two notes can be used to embed watermark into U and V components without visible distortion of the image.

3. Experimental results

In this section, several experiments are performed to verify the effectiveness when incorporating our proposed two notes in SVD-based watermakring scheme. Two gray images with size 512×512, say Lena and peppers, are used as the original images. As shown in Fig. 2, the binary image with size 32×32 is used as a watermark.

Table 1 shows the PSNR performance under different thresholds. The subscripts U and UV in the first column of Table 1 denote that the watermark is embedded into U component only and into both U and V

Fig. 2. The watermark.

Table 1
PSNR performance under different thresholds

Images	$T = 0.002$	$T = 0.012$	$T = 0.02$	$T = 0.04$
Lena_U	45.38	40.12	36.58	31.24
Lena_{UV}	43.59	38.69	34.92	29.88
Peppers_U	44.65	39.34	35.94	30.22
Peppers_{UV}	43.05	37.96	34.28	28.72

Table 2
Error rate under three attacks with $T = 0.012$

Images	JPEG QF = 70	Noise 5%	Cropping 25%
Lena_U	0.072	0.061	0.059
Lena_V	0.089	0.076	0.071
Peppers_U	0.044	0.049	0.042
peppers_V	0.068	0.071	0.077

components of the original image, respectively. Larger threshold value results in watermarked image with lower PSNR but makes the embedded watermark more robust. For the case of $T = 0.012$, we apply different attacks to the watermarked images to assess robustness. The attacks include the JPEG compression with a quality factor (QF) 70, an addition of 5% Gaussian noise, and a cropping of one–fourth of the upper left area of the watermarked images. The error rate between the extracted watermark and the original watermark is used to assess the robustness of the embedded watermark. The error rates of the above three attacks are shown in Table 2. The subscripts U and V in the first column of Table 2 denote the watermark extracted from the U component and the V component, respectively. Results of Tables 1 and 2 show that the proposed two notes can increase the invisibility and the capacity of the SVD-based watermarking scheme by embedding the watermark into U and V components of the SVD.

4. Conclusion

Our proposed two notes have been shown to increase the invisibility and capacity of the SVD-based watermarking scheme. Instead of embedding the watermark into the U component proposed by Chang et al., from the theoretical analysis, our proposed notes give a guideline to embed the watermark into both U and V components of the images. Experimental results demonstrate that the proposed two notes have both the PSNR and the capacity advantages of the SVD-based watermarking scheme.

Acknowledgements

Supported by National Council of Science of ROC under contracts NSC94-3114-P-001-001Y, NSC94-3114-P-011-001, and NSC94-2213-E-011-042.

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