Correspondence

The Complex Householder Transform

Kuo-Liang Chung and Wen-Ming Yan

***Abstract*— In this correspondence, a straightforward derivation for a complex Householder transform is given. It needs fewer complex operations when compared with the previous results by Venkaiah *et al.* and Xia and Suter. We also investigate applying our result to the derivation of a hyperbolic Householder transform.**

I. Introduction

The Householder transform is very useful in matrix computations and signal processing. In n-dimensional real vector space R^n for reflecting a vector $\mathbf{c}$ $(\in R^n)$ to $\mathbf{d}$ $(\in R^n)$, the Householder transform [1]–[2] is given by

$$\mathbf{H} = \mathbf{I} - 2\frac{\mathbf{v}\mathbf{v}^{\mathbf{T}}}{\mathbf{v}^{\mathbf{T}}\mathbf{v}} \tag{1}$$

where $\mathbf{T}$ denotes the transpose, and $\mathbf{v} = \mathbf{c} - \mathbf{d}$; it satisfies $\mathbf{Hc} = \mathbf{d}$ and $\|\mathbf{c}\| = \|\mathbf{d}\|$, where $\| \bullet \|$ denotes the vector norm. The Householder transform in (1) can be applied in n-dimensional complex vector space C^n if it satisfies $\mathbf{c}^*\mathbf{d} = \mathbf{d}^*\mathbf{c}$, where $\mathbf{c}, \mathbf{d} \in C^n$ [5], and $*$ denotes the transpose and complex conjugate. That is, we have $\mathbf{H} = \mathbf{I} - 2\frac{\mathbf{v}\mathbf{v}^*}{\mathbf{v}^*\mathbf{v}}$ if the above commutative law holds. In fact, $\mathbf{c}^*\mathbf{d} = \mathbf{d}^*\mathbf{c}$ is not true in general. In some special situations, in $\mathbf{C^n}$, some methods have been presented to apply the usual Householder transform [3]–[5].

Venkaiah *et al.* [5] extended (1) to C^n without any restriction. They first assumed that

$$\mathbf{H} = \mathbf{I} - \left(1 + \frac{\mathbf{a}^*\mathbf{z}}{\mathbf{z}^*\mathbf{a}}\right)\frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{z}} \tag{2}$$

where $\mathbf{a}, \mathbf{b} \in C^n$ and $\mathbf{z} = \mathbf{a} - \mathbf{b}$, and then, it was verified that $\mathbf{Ha} = \mathbf{b}$ and that $\mathbf{H}$ is unitary. Recently, Xia and Suter [6] proved the necessary part of the Householder transform [5]. If $\mathbf{a}^*\mathbf{a} \neq \mathbf{a}^*\mathbf{b}$, they first assumed that

$$\mathbf{H} = \mathbf{I} - (1 + y)\frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{z}} \tag{3}$$

where y is a complex number, and then, it was shown that $y = -\frac{\mathbf{z}^*\mathbf{b}}{\mathbf{z}^*\mathbf{a}}$.

In this correspondence, a complex Householder transform

$$\mathbf{H} = \mathbf{I} - \frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{a}} \tag{4}$$

is given. Not only the transform is shown by a straightforward derivation, but it also needs fewer complex operations when compared with the previous results by Venkaiah *et al.* [5] and Xia and Suter [6]. We also investigate applying our result to the derivation of a hyperbolic Householder transform.

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II. The Complex Householder Transform

Let $x\mathbf{w} = \mathbf{z} = \mathbf{a} - \mathbf{b} \neq \mathbf{0}$, where x is a positive real value, and $\mathbf{w}$ is a unit vector in C^n, i.e., $\|\mathbf{w}\| = 1$. Before deriving the complex Householder transform, we need the following two Lemmas.

Lemma 1: $x = \mathbf{w}^*\mathbf{a} + \mathbf{a}^*\mathbf{w}$.

Proof: Since the complex Householder transform must satisfy the two assumptions $\mathbf{Ha} = \mathbf{b}$ and $\|\mathbf{a}\| = \|\mathbf{b}\|$, it yields

$$\begin{aligned}\mathbf{a}^*\mathbf{a} = \mathbf{b}^*\mathbf{b} &= (\mathbf{a} - x\mathbf{w})^*(\mathbf{a} - x\mathbf{w})\\ &= \mathbf{a}^*\mathbf{a} - x\mathbf{w}^*\mathbf{a} - x\mathbf{a}^*\mathbf{w} + x^2\mathbf{w}^*\mathbf{w}.\end{aligned}$$

We then have $x = \mathbf{w}^*\mathbf{a} + \mathbf{a}^*\mathbf{w}$. □

Lemma 2: $x^2 = \mathbf{z}^*\mathbf{a} + \mathbf{a}^*\mathbf{z} = \mathbf{z}^*\mathbf{z}$.

Proof: From $\mathbf{z} = x\mathbf{w}$ and Lemma 1, we have

$$\begin{aligned}x^2 &= x(\mathbf{w}^*\mathbf{a} + \mathbf{a}^*\mathbf{w}) = x\mathbf{w}^*\mathbf{a} + \mathbf{a}^*x\mathbf{w} = \mathbf{z}^*\mathbf{a} + \mathbf{a}^*\mathbf{z}\\ &= (\mathbf{a} - \mathbf{b})^*\mathbf{a} + \mathbf{a}^*(\mathbf{a} - \mathbf{b}) = \mathbf{a}^*\mathbf{a} - \mathbf{b}^*\mathbf{a} + \mathbf{a}^*\mathbf{a} - \mathbf{a}^*\mathbf{b}\\ &= \mathbf{b}^*\mathbf{b} - \mathbf{b}^*\mathbf{a} + \mathbf{a}^*\mathbf{a} - \mathbf{a}^*\mathbf{b} = -\mathbf{b}^*(\mathbf{a} - \mathbf{b}) + \mathbf{a}^*(\mathbf{a} - \mathbf{b})\\ &= (\mathbf{a} - \mathbf{b})^*(\mathbf{a} - \mathbf{b}) = \mathbf{z}^*\mathbf{z}.\end{aligned}$$ □

The main result is shown in Theorem 1.

Theorem 1: The complex Householder transform is

$$\mathbf{H} = \mathbf{I} - \frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{a}}.$$

Proof: By Lemma 1 and 2, we have

$$\begin{aligned}\mathbf{b} &= \mathbf{a} - x\mathbf{w} = \mathbf{a} - (\mathbf{w}^*\mathbf{a} + \mathbf{a}^*\mathbf{w})\mathbf{w}\\ &= \mathbf{a} - \left(\frac{\mathbf{z}^*\mathbf{a}}{x} + \frac{\mathbf{a}^*\mathbf{z}}{x}\right)\frac{\mathbf{z}}{x} = \mathbf{a} - \frac{(\mathbf{z}^*\mathbf{a})\mathbf{z}}{x^2} - \frac{(\mathbf{a}^*\mathbf{z})\mathbf{z}}{x^2}\\ &= \mathbf{a} - \frac{(\mathbf{z}^*\mathbf{a})\mathbf{z}}{\mathbf{z}^*\mathbf{z}} - \frac{(\mathbf{a}^*\mathbf{z})\mathbf{z}}{\mathbf{z}^*\mathbf{z}} = \mathbf{a} - \frac{\mathbf{z}(\mathbf{z}^*\mathbf{a})}{\mathbf{z}^*\mathbf{z}} - \frac{(\mathbf{a}^*\mathbf{z})\mathbf{z}}{\mathbf{z}^*\mathbf{z}}.\end{aligned}$$

Putting $\mathbf{y}^*\mathbf{a}$ into the denominator and numerator of the third term in (5), respectively, we get

$$\begin{aligned}\mathbf{b} &= \mathbf{a} - \frac{\mathbf{z}\mathbf{z}^*}{(\mathbf{z}^*\mathbf{z})}\mathbf{a} - \frac{(\mathbf{a}^*\mathbf{z})\mathbf{z}\mathbf{y}^*}{(\mathbf{y}^*\mathbf{a})(\mathbf{z}^*\mathbf{z})}\mathbf{a}\\ &= \left(\mathbf{I} - \frac{\mathbf{z}\mathbf{z}^*}{(\mathbf{z}^*\mathbf{z})} - \frac{(\mathbf{a}^*\mathbf{z})\mathbf{z}\mathbf{y}^*}{(\mathbf{y}^*\mathbf{a})(\mathbf{z}^*\mathbf{z})}\right)\mathbf{a} = \mathbf{Ha}\end{aligned}$$

where the complex vector $\mathbf{y}$ will be determined later. Since we expect $\mathbf{H}$ to be unitary, i.e.,

$$\mathbf{H}^*\mathbf{H} = \mathbf{I}$$

where

$$\begin{aligned}\mathbf{H}^* &= \mathbf{I} - \frac{\mathbf{z}\mathbf{z}^*}{(\mathbf{z}^*\mathbf{z})} - \frac{(\mathbf{z}^*\mathbf{a})\mathbf{y}\mathbf{z}^*}{(\mathbf{a}^*\mathbf{y})(\mathbf{z}^*\mathbf{z})}\\ &= \mathbf{I} - \left(\frac{\mathbf{z}}{(\mathbf{z}^*\mathbf{z})} + \frac{(\mathbf{z}^*\mathbf{a})\mathbf{y}}{(\mathbf{a}^*\mathbf{y})(\mathbf{z}^*\mathbf{z})}\right)\mathbf{z}^* = \mathbf{I} - \mathbf{u}\mathbf{z}^*\end{aligned}$$

and

$$\mathbf{u} = \frac{\mathbf{z}}{(\mathbf{z}^*\mathbf{z})} + \frac{(\mathbf{z}^*\mathbf{a})\mathbf{y}}{(\mathbf{a}^*\mathbf{y})(\mathbf{z}^*\mathbf{z})}.$$

Here, $\mathbf{u} \neq 0$ because of $\mathbf{a} \neq \mathbf{b}$, i.e., $\mathbf{H}, \mathbf{H}^* \neq \mathbf{I}$. Therefore, we have

$$\mathbf{H}^*\mathbf{H} = (\mathbf{I} - \mathbf{u}\mathbf{z}^*)(\mathbf{I} - \mathbf{z}\mathbf{u}^*) = \mathbf{I} - \mathbf{u}\mathbf{z}^* - \mathbf{z}\mathbf{u}^* + \mathbf{u}\mathbf{z}^*\mathbf{z}\mathbf{u}^*.$$

If there exists a complex vector $\mathbf{q}$ such that $\mathbf{u}^*\mathbf{q} = 0$ and $\mathbf{z}^*\mathbf{q} \neq 0$, we have

$$\mathbf{H}^*\mathbf{H}\mathbf{q} = (\mathbf{I} - \mathbf{u}\mathbf{z}^* - \mathbf{z}\mathbf{u}^* + \mathbf{u}\mathbf{z}^*\mathbf{z}\mathbf{u}^*)\mathbf{q} = \mathbf{q} - \mathbf{u}(\mathbf{z}^*\mathbf{q}) \neq \mathbf{q}.$$

It is a contradiction because of

$$\mathbf{H}^*\mathbf{H} \neq \mathbf{I}.$$

Hence, it is impossible that any vector $\mathbf{q}$ exist such that $\mathbf{u}^*\mathbf{q} = 0$ and $\mathbf{z}^*\mathbf{q} \neq 0$. That is, there exists a complex number t such that $\mathbf{u} = t\mathbf{z}$. From

$$\mathbf{u} = \frac{\mathbf{z}}{(\mathbf{z}^*\mathbf{z})} + \frac{(\mathbf{z}^*\mathbf{a})\mathbf{y}}{(\mathbf{a}^*\mathbf{y})(\mathbf{z}^*\mathbf{z})}$$

there exists a complex number t' such that $\mathbf{y} = t'\mathbf{z}$. Hence, from $\mathbf{H} = \mathbf{I} - \mathbf{z}^*\mathbf{u}$, it follows that

$$\begin{aligned}\mathbf{H} &= \mathbf{I} - \frac{\mathbf{z}\mathbf{z}^*}{(\mathbf{z}^*\mathbf{z})} - \frac{(\mathbf{a}^*\mathbf{z})\mathbf{z}\mathbf{y}^*}{(\mathbf{y}^*\mathbf{a})(\mathbf{z}^*\mathbf{z})} = \mathbf{I} - \frac{\mathbf{z}\mathbf{z}^*}{(\mathbf{z}^*\mathbf{z})} - \frac{(\mathbf{a}^*\mathbf{z})\mathbf{z}(t'\mathbf{z})^*}{((t'\mathbf{z})^*\mathbf{a})(\mathbf{z}^*\mathbf{z})}\\ &= \mathbf{I} - \frac{\mathbf{z}\mathbf{z}^*}{(\mathbf{z}^*\mathbf{z})} - \frac{(\mathbf{a}^*\mathbf{z})\mathbf{z}\mathbf{z}^*}{(\mathbf{z}^*\mathbf{a})(\mathbf{z}^*\mathbf{z})} = \mathbf{I} - \left(1 + \frac{\mathbf{a}^*\mathbf{z}}{\mathbf{z}^*\mathbf{a}}\right)\frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{z}}\\ &= \mathbf{I} - \frac{\mathbf{z}^*\mathbf{a} + \mathbf{a}^*\mathbf{z}}{\mathbf{z}^*\mathbf{a}}\frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{z}}. \qquad (5)\end{aligned}$$

By Lemma 2, (6) can be written as

$$\mathbf{H} = \mathbf{I} - \frac{\mathbf{z}^*\mathbf{z}}{\mathbf{z}^*\mathbf{a}}\frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{z}} = \mathbf{I} - \frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{a}}.$$

Therefore, we obtain

$$\mathbf{H} = \mathbf{I} - \frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{a}} \qquad (6)$$

which satisfies $\mathbf{H}\mathbf{a} = \mathbf{b}$. In addition, $\mathbf{H}$ is unitary since we have

$$\begin{aligned}\mathbf{H}^*\mathbf{H} &= \left(\mathbf{I} - \frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{a}}\right)^*\left(\mathbf{I} - \frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{a}}\right)\\ &= \mathbf{I} - \left(\frac{1}{\mathbf{a}^*\mathbf{z}} + \frac{1}{\mathbf{z}^*\mathbf{a}} - \frac{\mathbf{z}^*\mathbf{z}}{(\mathbf{z}^*\mathbf{a})(\mathbf{a}^*\mathbf{z})}\right)\mathbf{z}\mathbf{z}^*\\ &= \mathbf{I} - \frac{\mathbf{z}^*\mathbf{a} + \mathbf{a}^*\mathbf{z} - \mathbf{z}^*\mathbf{z}}{(\mathbf{z}^*\mathbf{a})(\mathbf{a}^*\mathbf{z})}\mathbf{z}\mathbf{z}^* = \mathbf{I}. \qquad \Box\end{aligned}$$

From $\mathbf{z}^*\mathbf{a} = (\mathbf{a} - \mathbf{b})^*\mathbf{a} = \mathbf{a}^*\mathbf{a} - \mathbf{b}^*\mathbf{a} = \mathbf{b}^*\mathbf{b} - \mathbf{b}^*\mathbf{a} = \mathbf{b}^*(-\mathbf{z})$, we have $\mathbf{z}^*\mathbf{a} = -\mathbf{b}^*\mathbf{z}$. Thus, the Householder transform of (7) can also be written as

$$\mathbf{H} = \mathbf{I} + \frac{\mathbf{z}\mathbf{z}^*}{\mathbf{b}^*\mathbf{z}}.$$

III. Application to Hyperbolic Householder Transform

In this section, the result described in Section II will be applied to derive the hyperbolic Householder transform [4]. Let Φ be a diagonal matrix with diagonal entries $+1$ and -1. Suppose it satisfies

$$\mathbf{a}^*\Phi\mathbf{a} = \mathbf{b}^*\Phi\mathbf{b}$$
$$\Phi\mathbf{a} \neq \mathbf{b}$$

where $\mathbf{a}, \mathbf{b} \in C^n$. In the hyperbolic Householder transform, we want to find a hypernormal matrix $\mathbf{H}$ such that $\mathbf{H}\mathbf{a} = \mathbf{b}$ and $\mathbf{H}^*\Phi\mathbf{H} = \Phi$.

Let $\mathbf{H} = \Phi - \mathbf{u}\mathbf{v}^*$, where $\mathbf{u}$ and $\mathbf{v}$ will be determined later. Because of

$$\mathbf{H}\mathbf{a} = (\Phi - \mathbf{u}\mathbf{v}^*)\mathbf{a} = \Phi\mathbf{a} - \mathbf{u}\mathbf{v}^*\mathbf{a} = \mathbf{b}$$

we have $\Phi\mathbf{a} - \mathbf{b} = \mathbf{u}(\mathbf{v}^*\mathbf{a})$. Since $\Phi\mathbf{a} \neq \mathbf{b}$, it follows that $\mathbf{v}^*\mathbf{a} \neq 0$, $\mathbf{u} \neq 0$, $\mathbf{v} \neq 0$, and

$$\mathbf{u} = \frac{\mathbf{z}}{\mathbf{v}^*\mathbf{a}}$$

where $\mathbf{z} = \Phi\mathbf{a} - \mathbf{b}$. Therefore, it yields

$$\mathbf{H} = \Phi - \frac{\mathbf{z}}{\mathbf{v}^*\mathbf{a}}\mathbf{v}^*.$$

Consider

$$\begin{aligned}\mathbf{H}^*\Phi\mathbf{H} &= \left(\Phi - \frac{\mathbf{z}}{\mathbf{v}^*\mathbf{a}}\mathbf{v}^*\right)^*\Phi\left(\Phi - \frac{\mathbf{z}}{\mathbf{v}^*\mathbf{a}}\mathbf{v}^*\right)\\ &= \Phi - \frac{\mathbf{v}\mathbf{z}^*}{\mathbf{a}^*\mathbf{v}} - \frac{\mathbf{z}\mathbf{v}^*}{\mathbf{v}^*\mathbf{a}} + \frac{\mathbf{v}\mathbf{z}^*\Phi\mathbf{z}\mathbf{v}^*}{(\mathbf{a}^*\mathbf{v})(\mathbf{v}^*\mathbf{a})}.\end{aligned}$$

If there exists a complex vector $\mathbf{q}$ such that $\mathbf{v}^*\mathbf{q} = 0$ and $\mathbf{z}^*\mathbf{q} \neq 0$, we have

$$\begin{aligned}\mathbf{H}^*\Phi\mathbf{H}\mathbf{q} &= \left(\Phi - \frac{\mathbf{v}\mathbf{z}^*}{\mathbf{a}^*\mathbf{v}} - \frac{\mathbf{z}\mathbf{v}^*}{\mathbf{v}^*\mathbf{a}} + \frac{\mathbf{v}\mathbf{z}^*\Phi\mathbf{z}\mathbf{v}^*}{(\mathbf{a}^*\mathbf{v})(\mathbf{v}^*\mathbf{a})}\right)\mathbf{q}\\ &= \Phi\mathbf{q} - \frac{\mathbf{v}}{\mathbf{a}^*\mathbf{v}}\mathbf{z}^*\mathbf{q} \neq \Phi\mathbf{q}.\end{aligned}$$

It is a contradiction because of

$$\mathbf{H}^*\Phi\mathbf{H} = \Phi.$$

Hence, there cannot exist a vector $\mathbf{q}$ such that $\mathbf{v}^*\mathbf{q} = 0$ and $\mathbf{z}^*\mathbf{q} \neq 0$. We then have $\mathbf{v} = t\mathbf{z}$ for some complex number $t \neq 0$.

Therefore, we have

$$\mathbf{H} = \Phi - \frac{\mathbf{z}}{\mathbf{v}^*\mathbf{a}}\mathbf{v}^* = \Phi - \frac{\mathbf{z}}{\mathbf{z}^*\mathbf{a}}\mathbf{z}^*.$$

The matrix $\mathbf{H}$ is a hypernormal matrix since

$$\begin{aligned}\mathbf{H}^*\Phi\mathbf{H} &= \left(\Phi - \frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{a}}\right)^*\Phi\left(\Phi - \frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{a}}\right)\\ &= \Phi - \left(\frac{1}{\mathbf{a}^*\mathbf{z}} + \frac{1}{\mathbf{z}^*\mathbf{a}} - \frac{\mathbf{z}^*\Phi\mathbf{z}}{(\mathbf{z}^*\mathbf{a})(\mathbf{a}^*\mathbf{z})}\right)\mathbf{z}\mathbf{z}^*\\ &= \Phi - \frac{\mathbf{z}^*\mathbf{a} + \mathbf{a}^*\mathbf{z} - \mathbf{z}^*\Phi\mathbf{z}}{(\mathbf{z}^*\mathbf{a})(\mathbf{a}^*\mathbf{z})}\mathbf{z}\mathbf{z}^* = \Phi.\end{aligned}$$

From

$$\begin{aligned}\mathbf{z}^*\mathbf{a} + \mathbf{a}^*\mathbf{z} &= (\Phi\mathbf{a} - \mathbf{b})^*\mathbf{a} + \mathbf{a}^*(\Phi\mathbf{a} - \mathbf{b})\\ &= \mathbf{a}^*\Phi\mathbf{a} - \mathbf{b}^*\mathbf{a} + \mathbf{a}^*\Phi\mathbf{a} - \mathbf{a}^*\mathbf{b}\end{aligned}$$

and

$$\begin{aligned}\mathbf{z}^*\Phi\mathbf{z} &= (\Phi\mathbf{a} - \mathbf{b})^*\Phi(\Phi\mathbf{a} - \mathbf{b}) = (\mathbf{a}^*\Phi - \mathbf{b}^*)\Phi(\Phi\mathbf{a} - \mathbf{b})\\ &= \mathbf{a}^*\Phi\mathbf{a} - \mathbf{a}^*\mathbf{b} - \mathbf{b}^*\mathbf{a} + \mathbf{b}^*\Phi\mathbf{b}\end{aligned}$$

we obtain $\mathbf{z}^*\mathbf{a} + \mathbf{a}^*\mathbf{z} = \mathbf{z}^*\Phi\mathbf{z}$ because of $\mathbf{a}^*\Phi\mathbf{a} = \mathbf{b}^*\Phi\mathbf{b}$

Finally, $\mathbf{H}\mathbf{a} = \mathbf{b}$ can be verified as

$$\mathbf{H}\mathbf{a} = \left(\Phi - \frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{a}}\right)\mathbf{a} = \Phi\mathbf{a} - \mathbf{z} = \mathbf{b}.$$

The desired hypernormal matrix becomes

$$\mathbf{H} = \Phi - \frac{\mathbf{z}\mathbf{z}^*}{\mathbf{z}^*\mathbf{a}}.$$

IV. Discussions and Conclusions

Looking at the guessed forms of the complex Householder transform in [5] and [6] and our form of the Householder transform [see (7)], our form uses fewer complex operations when compared with the guessed forms [5], [6] since both previous forms [5], [6] need three complex inner-product operations and one complex outer-product operation, respectively, whereas our transform only needs one complex inner-product operation and one complex outer-product operation. From $\mathbf{a}^*\mathbf{z} = \mathbf{a}^*(\mathbf{a} - \mathbf{b}) = \mathbf{a}^*\mathbf{a} - \mathbf{a}^*\mathbf{b} = \mathbf{b}^*\mathbf{b} - \mathbf{a}^*\mathbf{b} = -\mathbf{z}^*\mathbf{b}$, we have $\mathbf{a}^*\mathbf{z} = -\mathbf{z}^*\mathbf{b}$. Thus, the guessed form in [6] [see (3)] is equal to the guessed form in [5] [see (2)]. Surprisingly interestingly, the intermediate form [see (6)] in our direct derivation is equal to the guessed form in [5]. It means that by Lemma 2, the guessed forms in [5] and [6] can be simplified into our final form [see (7)].

The significance of the Householder transform is due to its popular use in many fields of matrix computations and signal processing. The main contributions of this paper are threefold: First, we give the direct derivation of the complex Householder transform; second, we apply this derivation to derive the hyperbolic Householder transform directly; third, the previous guessed complex Householder transforms in [5] and [6] can be simplified by using our partial result, i.e., Lemma 2. In fact, according to the results of this correspondence, a block representation for products of hyperbolic Householder transform [7], which is very suitable for vector supercomputing, has also been derived.

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Extended Overlap-Add and -Save Methods for Multirate Signal Processing

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***Abstract*—The overlap-add method (OLA) and overlap-save method (OLS) are well known as efficient schemes for high-order FIR filtering. In this correspondence, new sampling rate conversion methods are proposed by extending the OLA and OLS and eliminating the redundancy caused by the conversion. First, for finite-duration sequences, a rate conversion with the DFT-domain approach is discussed. Then, using the result, the extended OLA and OLS are proposed for infinite-duration sequences. Last, the computational complexities of our proposed methods are shown.**

***Index Terms*—Fast Fourier transform (FFT), overlap-add/save method, sampling rate conversion.**

I. INTRODUCTION

Multirate signal processing is indispensable as a fundamental scheme for reduction of computation, improvement of processing precision, compression of information, etc. [1]. In this work, we consider efficiently converting the sampling rate of a digital signal. The conversion can be divided into two classes. One is a rate reduction, and the other is a rate increase. For the discussion without loss of generality, we deal with a sampling rate conversion by an arbitrary rational factor U/D, where D and U are integers.

A matter of serious concern on rate conversion is filtering since it accounts for most of the operations, and the filter characteristics determine the performance of the system. Hence, the redundant operation caused by the conversion should be avoided. Furthermore, it is of interest to efficiently perform the high-order filtering in terms of the characteristics. For these reasons, various techniques avoiding the redundancy and decreasing the complexity have been investigated. The polyphase decomposition [1] and the FFT-based interpolation method [2]–[4] represent those techniques. The complexities of the former, however, increase linearly with the tap length of the filter. On the other hand, the latter covers only a finite-duration data sequence, although the complexities are insensitive to the tap length because of the DFT-domain approach.

Therefore, in order to provide an efficient rate conversion technique, which can handle infinite-duration sequences with the DFT-domain approach, we propose to extend the overlap-add method (OLA) and overlap-save method (OLS) for FIR filtering to the sampling rate conversion.

II. SAMPLING RATE CONVERSION

In this section, we review sampling rate conversion and consider the DFT-domain approach, supposing that the input data sequence is of finite length.

A. Rate Conversion with the Time-Domain Approach

Fig. 1 indicates the basic structure of the sampling rate conversion system [1].

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